CONTENT MODULE 1. Mechanics and Thermodynamics of Biological Processes

TOPIC 1.A. Elements of Biomechanics

Biological tissues are complex in the structure and non-homogeneous in the composition. The structure and properties of biological tissues are determined by the functions they perform in living organisms. The biomechanics studies various forms of mechanical motion in living systems.

Exercise 1.1a. The main task of the mechanics is to determine the position of the body in space at any moment time. No matter what your interest in science, mechanics will be important for you - motion is a fundamental idea in all of science. Define the following terms and write down the answer:

| The point mass is |
|-------------------------|
| |
| Perfectly rigid body is |
| Translational motion is |
| Rotational motion is |
| The axis of rotation is |
| Trajectory is |
| Displacement is |



Fig. 1.1a. Circular motion of a point mass

Kinematic Characteristics of Translational and Rotational Motion

Exercise 1.2a. Fill in the Table 1.1a with the basic characteristics of motion:

| | Table | 1.1a |
|-------|-------|------|
| Units | of | |

| Characteristic | Definition | Formula | | Units of |
|-------------------------|----------------|------------|--------|-------------|
| | | | | measurement |
| The instantaneous | | v = | (1.1a) | |
| linear speed | | | | |
| The instantaneous | | $\omega =$ | (1.2a) | |
| angular velocity | | | | |
| The instantaneous | | <i>a</i> = | (1.3a) | |
| linear acceleration | | | | |
| The instantaneous | | £ = | (1.4a) | |
| angular acceleration | | | | |
| The acceleration if the | ≓ a∵τ | <i>a</i> = | (1.5a) | |
| body moves along the | | | | |
| curvilinear trajectory | \overline{n} | $a_t =$ | (1.6a) | |
| | | | | |
| | | $a_n =$ | (1.7a) | |
| | a(t) | " | | |
| | | | | |

Exercise 1.3a. Write down the linear and angular quantities relationship:

Problem 1.4a. The normal acceleration for the point that moves along a circular path with radius 9 m is given in the form $a_n = A + Bt + Ct^2$ (A = 1 m/s², B = 6 m/s³, C = 9 m/s⁴). Find 1) the tangential acceleration; 2) the distance covered by the point in the first 5 seconds (t₁); 3) total acceleration at the moment of time t₂ = 1 s.

<u>Data</u>:

Solution:

Exercise 1.5a. Fill in the Table 1.2a about motion types:

| Uniform motion | Uniformly variable motion |
|-------------------|------------------------------|
| $\epsilon = 0$ | $\varepsilon = \text{const}$ |
| ω = | ω = |
| $\varphi = (1.1)$ | a) $\varphi = (1.12a)$ |

With uniform circular motion, the concepts of period T, frequency v and angular frequency ω are used. They are related as follows:

| Τ= | (1.13a) |
|----|-----------|
| 1 | (1.1.5 a) |

$$\omega = (1.14a)$$

Problem 1.6a. Revolution rate of the wheel reduced from 300 to 180 rpm during one minute. Find 1) angular acceleration of the wheel; 2) the number of full revolutions during this time.

| <u>Data</u> : | Solution: |
|---------------|-----------|
| | |
| | |
| | |
| | |
| | |
| | |
| Answer: | |

Dynamics

Exercise 1.7a. The basis of the dynamics are three Newton's laws. They are a generalization of numerical observations, theoretical studies and experiments. For further consideration it is necessary to define the following terms and specify the unit of measurement:

The mass is

The momentum is_____

The force is_____

Exercise 1.8a. Fill in the Table 1.3a with the basic characteristics and equations of dynamics of translational and rotational motion:

| Rotational motion | | | | |
|---|--|--|--|--|
| Moment of inertia | | | | |
| Torque (1.16a) | | | | |
| Angular momentum (1.18a) | | | | |
| First Law | | | | |
| | | | | |
| (1.20a) | | | | |
| econd Law | | | | |
| (1.22a) | | | | |
| third law | | | | |
| (1.24a) | | | | |
| The law of conservation of angular momentum | | | | |
| (1.26a) | | | | |
| Work | | | | |
| (1.28a) | | | | |
| The kinetic energy | | | | |
| (1.30a) | | | | |
| rvation of energy | | | | |
| (1.32a) | | | | |
| ver | | | | |
| (1.34a) | | | | |
| | | | | |

Exercise 1.9a. Fill in the Table 1.4a with the moment of inertia of homogeneous rigid bodies of regular geometric shape relative to the centroidal axis.

Table 1.4a.

Table 1 3a

| Body | Moment of inertia |
|---|-------------------|
| A solid sphere of the radius R | (1.35a) |
| A solid hollow cylinder with the inner radius r and the | |
| external one R | |
| A thin-walled cylinder $(R = r)$ | (1.37a) |
| A solid cylinder $(r = 0)$ | (1.38a) |
| A thin rod of the length l | (1.39a) |
| The theorem of parallel axes | (1.40a) |

Problem 1.10a. The molecule has elastic collision with the container wall. Find the kinetic energy of the molecule after collision if it has mass $m = 4.65 \times 10^{-26}$ kg and its initial speed was v = 600 m/s.



Control questions

- 1. List the main characteristics of the motion.
- 2. What is the force? Define it.
- 3. What does characterizes the tangential component of the total acceleration? The normal (centripetal) component?
- 4. What is the difference between the concepts of energy and work?
- 5. What is the moment of inertia?
- 6. Is Newton's first law consequence of the second law? Why?

Individual assignments

- 1. The point moves along the circle of the radius R = 2 cm according to the equation $x = Ct^3$, where C = 0.01 cm/s³. Find normal and tangential acceleration of the point at the moment when its linear speed v = 0.3 m/s.
- 2. The number of revolutions of centrifuge rotor reaches $n = 2 \times 10^4$ rpm. After switching off of the engine the rotation is ceased in $t_1 = 8$ min. Find the angular acceleration ε , dependence of the angular displacement of the centrifuge on time $\varphi(t)$ and the number of full revolutions N made by the rotor before complete stopping. The motion should be considered as uniformly decelerated.
- 3. Find the moment of inertia of homogeneous rigid ring of the mass m = 1 kg relative to the centroidal axis. The inner radius of the ring r = 10 cm and the external one R = 30 cm.
- 4. Revolution rate of the figure skater is 6 revolutions per second. Find the change in the moment of inertia of the figure skater if after he press his arms to his body his revolution rate will be 18 revolutions per second.
- 5. The cooler starts rotating with the constant angle acceleration of 0.3 rad/s² and after 15 seconds it will have the angular momentum of 30 kg·m²/s. Find the kinetic energy of the cooler after 20 seconds after beginning of rotation.
- 6. The bullet of mass m = 10 g flies at a speed of v = 800 m/s, rotating about its longitudinal axis at a frequency v = 3000 s⁻¹. Find the total kinetic energy of the bullet. The bullet should be considered as a cylinder of diameter d = 8 mm.

TOPIC 1.B. Oscillatory Processes in Living Organisms. Bioacoustics Mechanical Vibrations

Many processes occurring in biological systems are periodical. They are observed in functional activity of the heart, lungs, stomach. Some processes in living organisms can be considered as oscillating processes e.g. the fluctuations of vessel's walls during propagation of the pulse wave, the change of air volume in the lungs, the vibrations of eardrums and others. To determine the norm or pathology of an organ, a graphic record of periodic processes that accompany its functional activity is usually analyzed. To solve such problems, it is necessary to know the common patterns which characterize all oscillatory processes regardless of their nature, and can only be described by mathematical equations.

Exercise 1.1b. Define the following terms and write down the answer:

Oscillations are_____

| The types of oscillati | ons (List): 1 | |
|------------------------|---------------|--|
| | 2 | |
| | 3 | |
| Damped oscillations | are | |
| Free oscillations are | | |
| | | |

Although the physical nature of oscillatory systems can vary significantly, various types of fluctuations can be described quantitatively by certain parameters.

Exercise 1.2b. Fill in the Table 1.1b with parameters describing oscillatory processes.

Table 1.1b

| Parameter | Symbol | Unit of measurement | Definition |
|--------------------------------------|--------|------------------------|------------|
| Displacement | | | |
| Amplitude | | | |
| Period | | | |
| Frequency | | | |
| Circular frequency (angular rate) | | | |
| The initial phase | | | |
| Wavelength | | | |

Characteristics of the oscillation process



Fig. 1.1b. Parameters describing oscillatory processes.

The differential equation of free undamped oscillations is:
$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$
. (1.1b)

The solution is:

$$x = A\cos(\omega_0 t + \varphi_0)$$
 or $x = A\sin(\omega_0 t + \varphi_0)$. (1.2b)

(1.6b)

The speed of an oscillating mass point is:

$$\upsilon = \frac{dx}{dt} = -A\omega_0 \sin(\omega_0 t + \phi_0) = -\upsilon_{\max} \sin(\omega_0 t + \phi_0) = \upsilon_{\max} \cos(\omega_0 t + \phi_0 + \pi/2), \quad (1.3b)$$

where
$$v_{\text{max}} = A \cdot \omega_0$$
 - amplitude of speed. (1.4b)

The acceleration of an oscillating mass point is:

$$a = \frac{d\nu}{dt} = -A\omega_0^2 \cos(\omega_0 t + \phi_0) = -a_{\max} \cos(\omega_0 t + \phi_0) = a_{\max} \cos(\omega_0 t + \phi_0 + \pi), \qquad (1.5b)$$

where $a_{\text{max}} = A \cdot \omega_0^2$ - amplitude of acceleration.

Exercise 1.3b. Energy of an oscillating point is (finish writing):

| - kinetic: $K=$ | (1.7b) |
|-----------------|--------|
| - potential: U= | (1.8b) |

$$- \text{ total}: E = K + U \tag{1.9b}$$

Exercise 1.4b. The attenuation rate is characterized by (finish writing):

| - decay factor: $\beta =$ | (1.10b) |
|--------------------------------------|---------|
| - logarithmic decrement: $\lambda =$ | (1.11b) |

Exercise 1.5b. Define the following terms and write down the answer:

Resonance is_____

The value of the resonant frequency: $\omega_{res} =$ (1.12b) Give examples of oscillatory systems.

Example of problem solution.

The point oscillates harmonically with a frequency f = 10 Hz. At the initial instant t = 0 the point has a maximum displacement $x_{max} = 1 mm$. Write down the oscillation equation for the point.

| <u>Data</u> : | Solution: | |
|--|---|---------|
| f = 10 Hz. | Equation for the point can be written as: | |
| $x_{\rm max}=1mm=10^{-3}m$ | $x = A\sin(\omega_0 t + \phi_0)$ | (1) |
| Equation -? | By definition, the amplitude: $A = x_{max}$. | |
| | The angular frequency ω associated with frequency | f as: |
| | $\omega = 2 \pi f .$ | (2) |
| | At time $t = 0$ formula (1) takes the form: | |
| | $x_{\max} = A\sin\phi_0$ | (3) |
| | where the initial phase: $\phi_0 = \arcsin(x_{\max} / A) = \arcsin 1$. | |
| | Phase changing on 2π does not change the state of oscil | llation |
| | point, so we can assume that: $\phi_0 = \pi / 2$ | (4) |
| | Taking into account (2) - (4) The equation takes the form: | |
| | $x = A\sin(2\pi f t + \phi_0)$, or $x = A\cos 2\pi f t$, | |
| | where $A = 1 mm = 10^{-3} m$, $f = 10 Hz$, $\phi_0 = \pi / 2$. | |
| Answer: $x = A\sin(2\pi f t + \phi_0)$ | or $x = A \cos 2\pi f t$ - the oscillation equation. | |

Problem 1.6b. The motion equation for the point is given in the form $x = 2\sin\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)$ cm. Find

the period of oscillations T, the maximum speed v_{max} and maximum acceleration a_{max} of the point.

Solution:

Answer:

Problem 1.7b. The point oscillates harmonically. The initial phase of oscillations $\phi_0 = 0$. Find relationship between kinetic and potential energy of the point at the moment of time $t = \frac{T}{12}$ s.



Mechanical Waves

The process of the vibrational motion propagation in a medium called a mechanical wave. This process can be described by changes of particle position in time and space.

Exercise 1.8b. Fill in the scheme (definition, equation):



Exercise 1.9b. Define the following terms and write down the answer: *Sound* is

Ultrasound is

Infrasound is_____

Give examples of ultrasound and infrasound.

Wave process is characterized by phase velocity, wavelength, frequency or period of oscillations. Write down how these components relate to each other:

(1.13b)

Exercise 1.10b. Fill in the table 1.2b.

Table 1.2b

| Characteristic | Symbol | Unit of | Definition (formula) | |
|-----------------|--------|-------------|--|--|
| | | measurement | | |
| Speed | | | $v = \dots $ | |
| Energy flux | | | $W = \dots $ | |
| Sound intensity | | | <i>I</i> =(1.16b) | |
| Sound pressure | | | $P = \dots $ | |

The physical characteristics of sound

Exercise 1.11b. Define the following terms, write down formulas and constants.

Sound intensity level (L_{dB}) is_____

The sound intensity level is given by:

$$L_{dB} = 10 \log_{10} \left(\frac{I}{I_0}\right) = 10 \lg(\frac{I}{I_0}) \text{ [dB]}.$$
 (1.18b)

*The threshold of hearing (I*₀) is_____

Value: $I_0 = 10^{-12} \text{ W} \cdot \text{m}^{-2}$ Threshold of pain (I_{max}) is

Value: $I_{\text{max}} = 10 \text{ W} \cdot \text{m}^{-2}$.

Exercise 1.12b. How we can rewrite expression (1.18b) using the sound pressure?

.....

(1.19b)

Problem 1.13b. Calculate what sound intensity level corresponds to intensity 2.0×10^{-7} W·m⁻²?



Fig. 1.2b. The relationship of loudness in phons to intensity level (in decibels) for persons with normal hearing. The curved lines are equal-loudness curves - all sounds on a given curve are perceived as equally loud. Phons and decibels are defined to be the same at 1000 Hz.

Exercise 1.14b. Calculate and enter to the Table 1.3b intensity of the following sound sources. Make a comparative analysis of different sound sources, draw conclusion.

Table 1.3b

| Source | Intensity, <i>I</i> , W·m ⁻² | Sound intensity level, L, dB |
|---------------------------|--|---------------------------------|
| Threshold of audibility | | 5 |
| Palpitation of the leaves | | 15 |
| Whisper | | 20 |
| Good dialogue | | 40 |

The intensities of different sound sources

| Traffic on the street | 65 |
|------------------------------------|-----|
| Vacuum cleaner | 80 |
| Big band | 98 |
| iPad at maximum volume level | 100 |
| The front rows at a rock concert | 115 |
| The threshold of pain | 130 |
| Take-off of an aircraft | 140 |
| Instant perforation of the eardrum | 160 |

Doppler Effect

The Doppler effect (or *the Doppler shift*) is the change in frequency or wavelength of a wave for an observer who is moving relative to the wave source.

If an observer and / or audio source is moving:

$$f_{observer} = \left(\frac{\upsilon \pm \upsilon_{observer}}{\upsilon \mp \upsilon_{source}}\right) f_{source}.$$
(1.20b)

v is speed of sound; v_{source} is speed of sound sources; $v_{observer}$ is speed of the observer; f_{source} is frequency emitted by the sound source; $f_{observer}$ is the frequency detected by observer.

In the numerator "+" sign is used when the observer moves in the direction of the sound source, and "-" - on the contrary. In the denominator sign "-" corresponds source movement toward the observer, and "+" - on the contrary.

Problem 1.15b. Two ambulance cars are moving towards each other at speeds of 20 and 10 m·s⁻¹. The first car beeps at a frequency of 800 Hz. Calculate the frequency of the signal that second ambulance car driver hears: a) before cars meetings; b) after cars meeting? The speed of sound is $348 \text{ m}\cdot\text{s}^{-1}$.

<u>Data</u>:

Solution:

Answer:

Exercise 1.16b. Feel in the Table 1.4b. Write down objective characteristics of sound that correspond to the given subjective characteristics.

Audio characteristics

| Subjective characteristics | Objective characteristics |
|----------------------------|----------------------------------|
| Pitch of a tone | |
| Timbre (tone color) | |
| Loudness (sound volume) | |
| | |

Control questions

- 1. List the main parameters of the wave process.
- 2. Units of measurement of sound intensity levels.
- 3. How does the intensity of the ultrasonic waves change as they pass through the biological objects?
- 4. Compare free, damped and forced oscillations. What are the similarities and the differences?
- 5. What type of oscillation is the heartbeats?
- 6. What is the difference between simple tone and the complex one?
- 7. What is the audiometry? In what cases we should use this research method?
- 8. Name the properties of ultrasonic waves.

Individual assignments

- 1. In the laboratory of laying house noise intensity level is 80 dB. To reduce the noise, it was decided to sheathe the walls of the laboratory with absorbent material that reduces sound intensity 1500 times. What is the level of noise intensity will be observed in the laboratory after renewal?
- 2. When walking the man's hand performs harmonic oscillations according to law $x = 17 \sin 1.6\pi t \ cm$. Determine the time of shifting of the hand from its equilibrium position to the position of maximum displacement.
- 3. The mass point performs simple harmonic oscillations. The initial displacement $x_0 = 4 cm$ and speed $v_0 = 10 cm \cdot s^{-1}$. Determine the amplitude A and the initial phase of oscillation φ_0 , if their period of oscillations T = 2 s.
- 4. The tuning fork produces a sound with frequency 400 Hz. Determine the maximum speed and acceleration of the end branches of the tuning fork if the amplitude is 0.2 mm.
- 5. The sound volume with frequency 200 Hz increased from 20 to 50 phon. How many times has increased the intensity of sound?
- 6. Determine the loudness of sound with the intensity level 60 dB, if the frequency equals to 50, 100, 800 and 7000 Hz.
- 7. The noise in the room of the laying house is 95 dB during the day and at night it decreases to 65 dB. How many times has increased the intensity of sound from night to day?
- 8. A mechanical milker working in the cattle house, produces a noise with intensity level 75 dB. Determine the noise intensity level of 3 simultaneously working mechanical milkers.