NATIONAL UNIVERSITY OF PHARMACY
Department of Educational and Information Technologies

BIOPHYSICS, PHYSICAL METHODS OF ANALYSIS

Lecture 1

Mechanical oscillations and waves.
Biophysics of muscle contraction.

Plan of the Lecture

1. Characteristics of periodic motion.
2. Simple harmonic motion (SHM).
3. Energy in SHM.
4. Damped oscillations.
5. Types of mechanical waves.
6. Types of muscles.
7. Muscle fiber, muscle cell.
8. Muscle Contraction.
9. Force-Velocity Relationship.
10. Hill Equation.

Purpose of the lecture is

- studying of mechanical oscillations and waves parameters. Their application in medical physics.
- to learn concepts related with the structure of of muscle fibers and the characteristics of its work.

The subject of the course “Biological Physics and Physical Methods of Analysis” is the knowledge of the physical processes occurring in biological environment, the impact of external factors on living organisms and physical methods of analysis used in pharmacy.

“Biological Physics and physical methods of analysis” is one of the fundamental general subjects that forms the theoretical basis for highly qualified specialists training for pharmacy. The studying of the course forms basic understanding of the general properties and the forms of motion, the most important physical laws that are fundamentals of mechanical, thermal, electrical, magnetic, spectral, polarization and other physical methods of investigation of various properties of drugs.

What causes periodic motion?

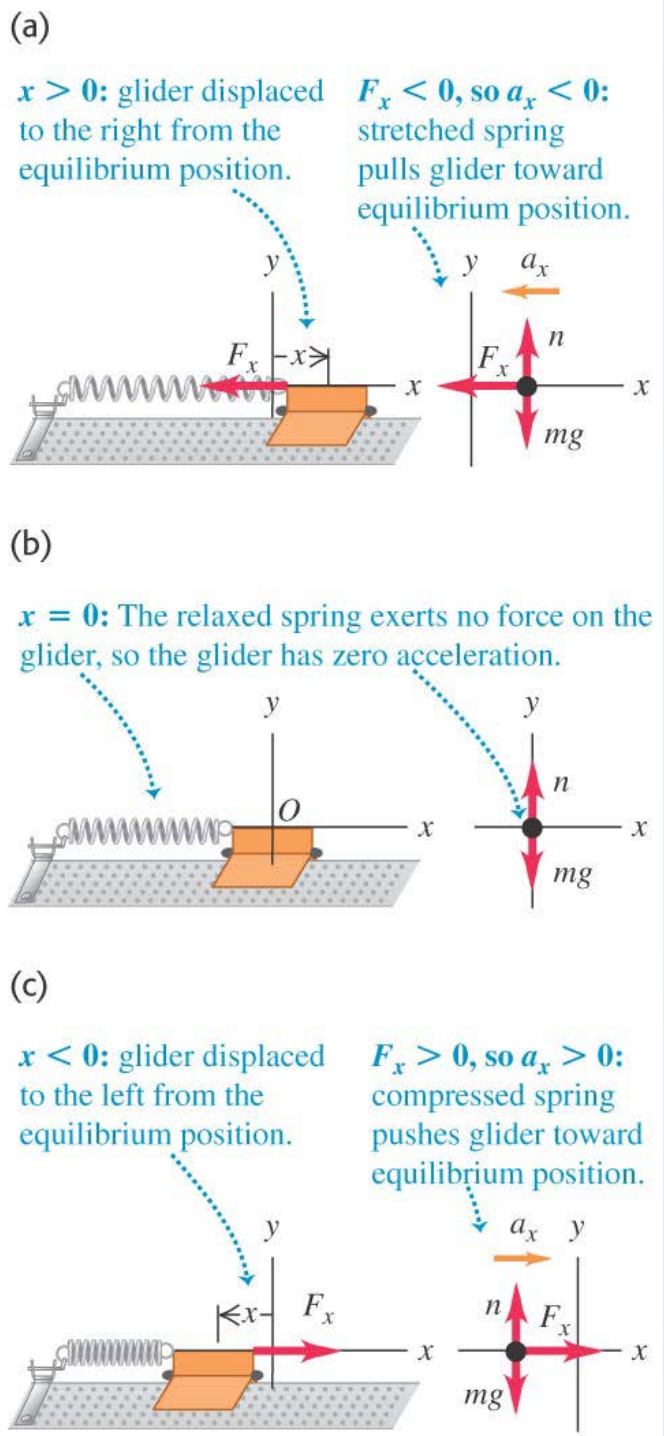
- If a body attached to a spring is displaced from its equilibrium position, the spring exerts a *restoring force* on it, which tends to restore the object to the equilibrium position. This force causes *oscillation* of the system, or *periodic motion*.

- Figure at the right illustrates the restoring force F_x.

$$F_x = -kx$$

$$ma = -kx$$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

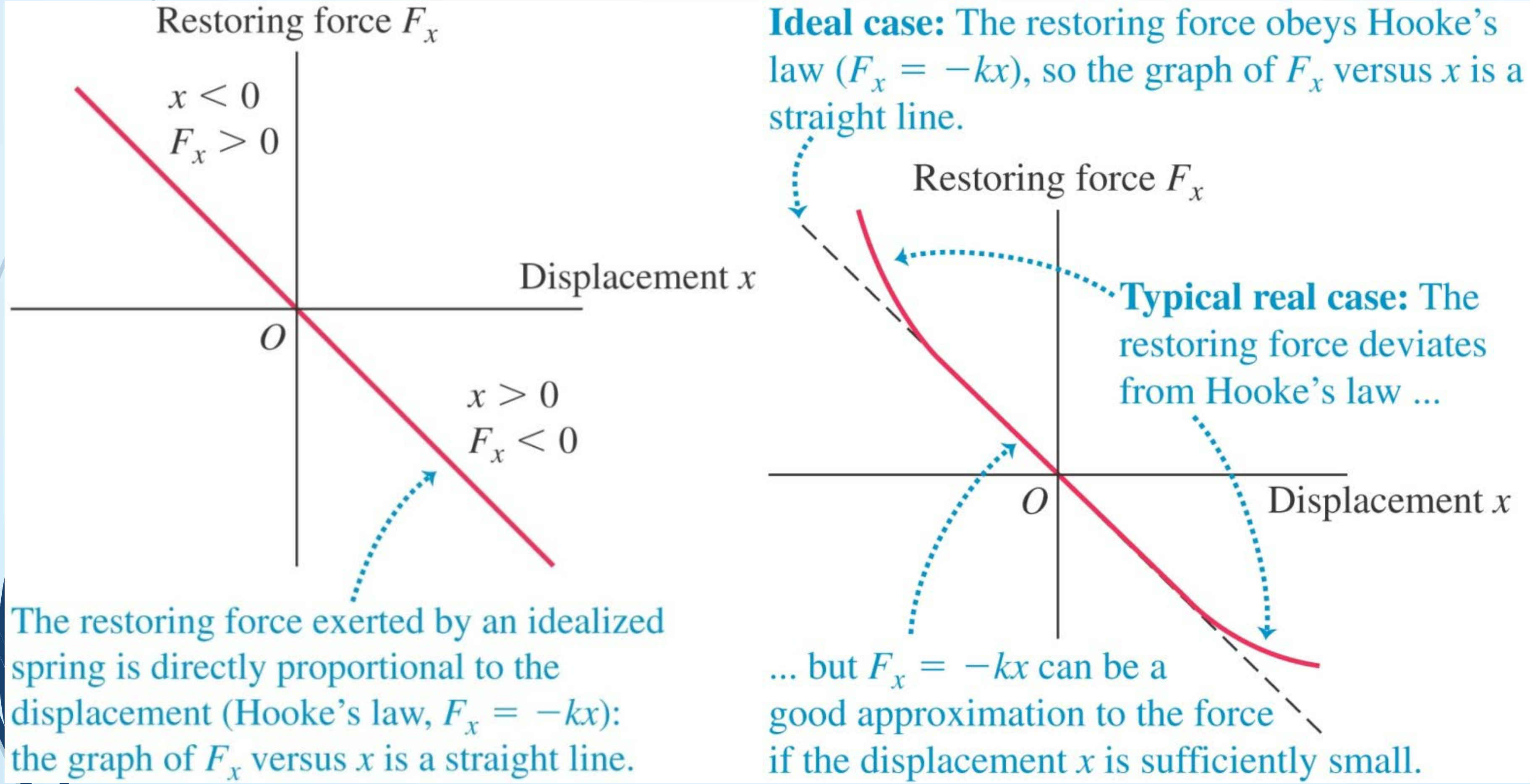


Characteristics of periodic motion

- The *amplitude*, A, is the maximum magnitude of displacement from equilibrium.
- The *period*, T, is the time for one cycle.
- The *frequency*, f, is the number of cycles per unit time.
- The *angular frequency*, ω, is 2π times the frequency: $\omega = 2\pi f$.
- The frequency and period are reciprocals of each other: $f = 1/T$ and $T = 1/f$.

Simple harmonic motion (SHM)

- When the restoring force is *directly proportional* to the displacement from equilibrium, the resulting motion is called *simple harmonic motion* (SHM).
- An ideal spring obeys Hooke's law, so the restoring force is $F_x = -kx$, which results in simple harmonic motion.



$$F_x = -kx$$

$$ma = -kx$$

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Characteristics of SHM

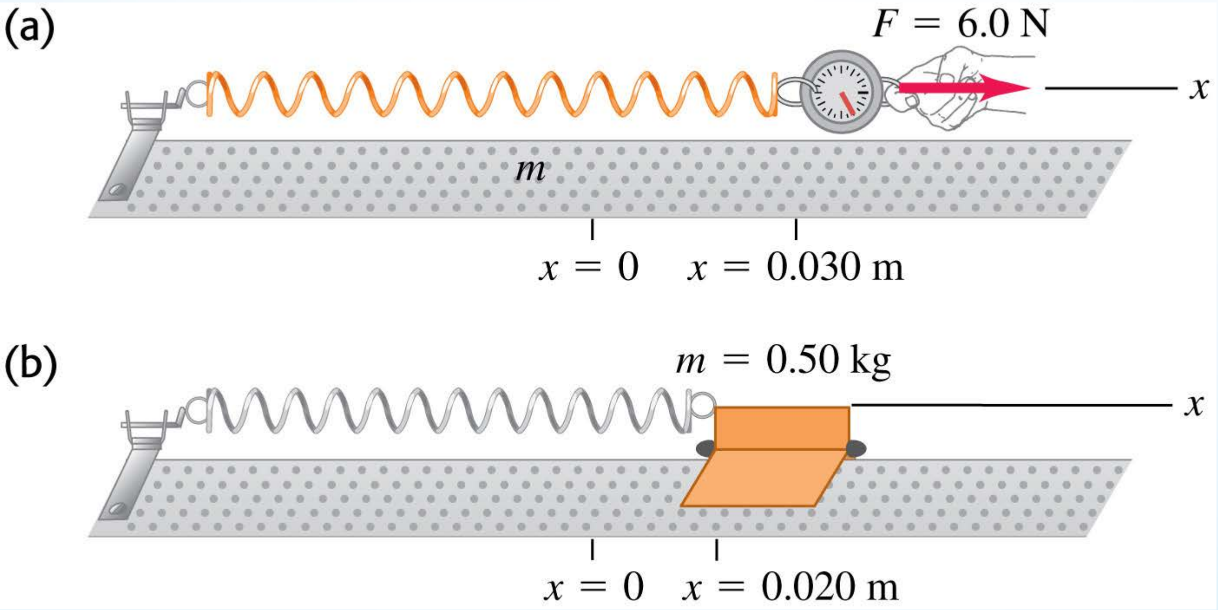
- For a body vibrating by an ideal spring:

$$\omega = \sqrt{\frac{k}{m}} \quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

- Follow Example below.

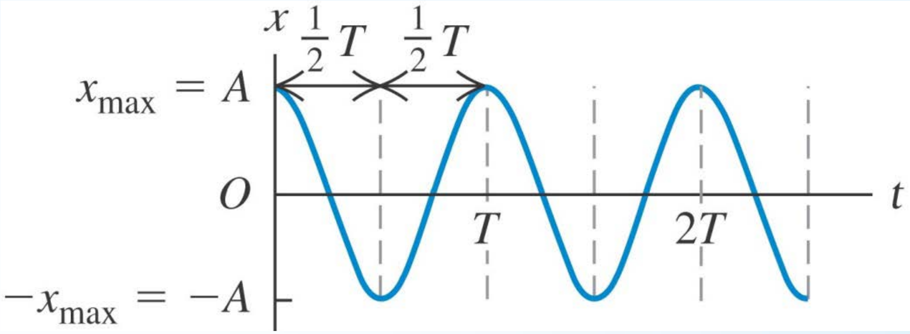
  (a) Find force constant *k* of the spring

  (b) Find angular frequency, frequency, and period of oscillation

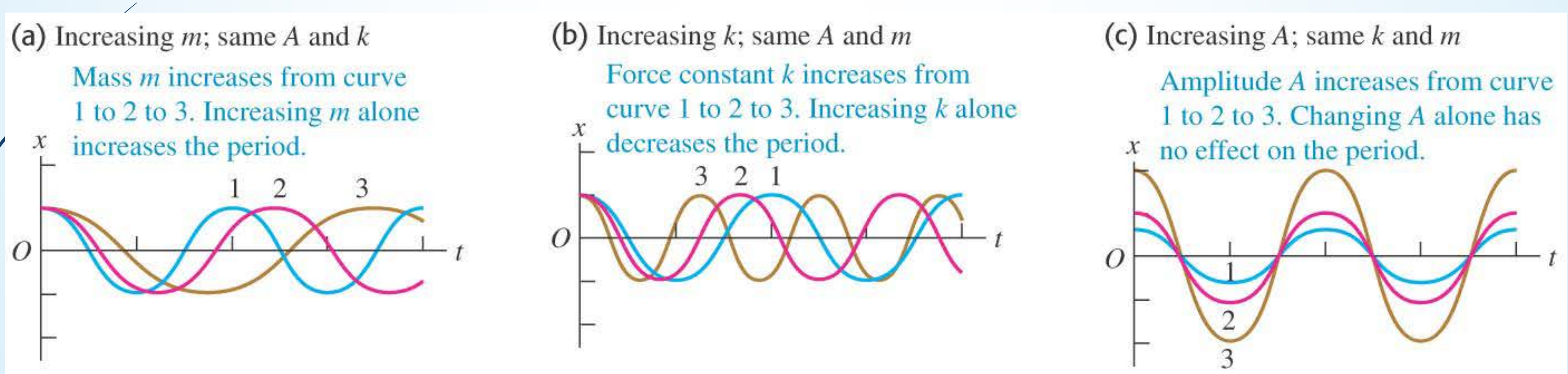


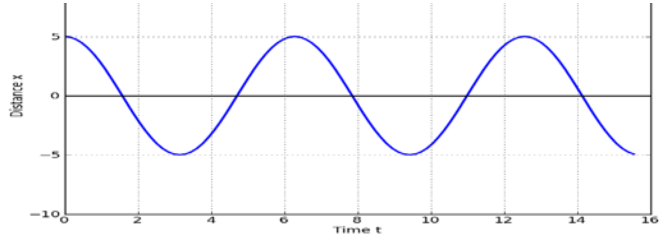
Displacement as a function of time in SHM

- The displacement as a function of time for SHM with phase angle ϕ is $x = A\cos(\omega t + \phi)$. (See Figure at right.)

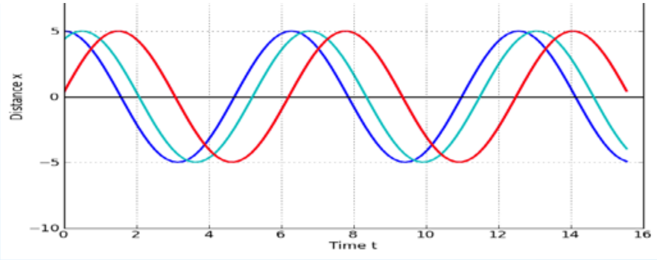


- Changing m, A, or k changes the graph of x versus t, as shown below.





change ϕ →



Displacement, Velocity and Acceleration

- The displacement as a function of time for SHM with phase angle ϕ is:

$$x(t) = A\cos(\omega t + \phi)$$

- As always, velocity is the time-derivative of displacement:

$$v_x(t) = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$$

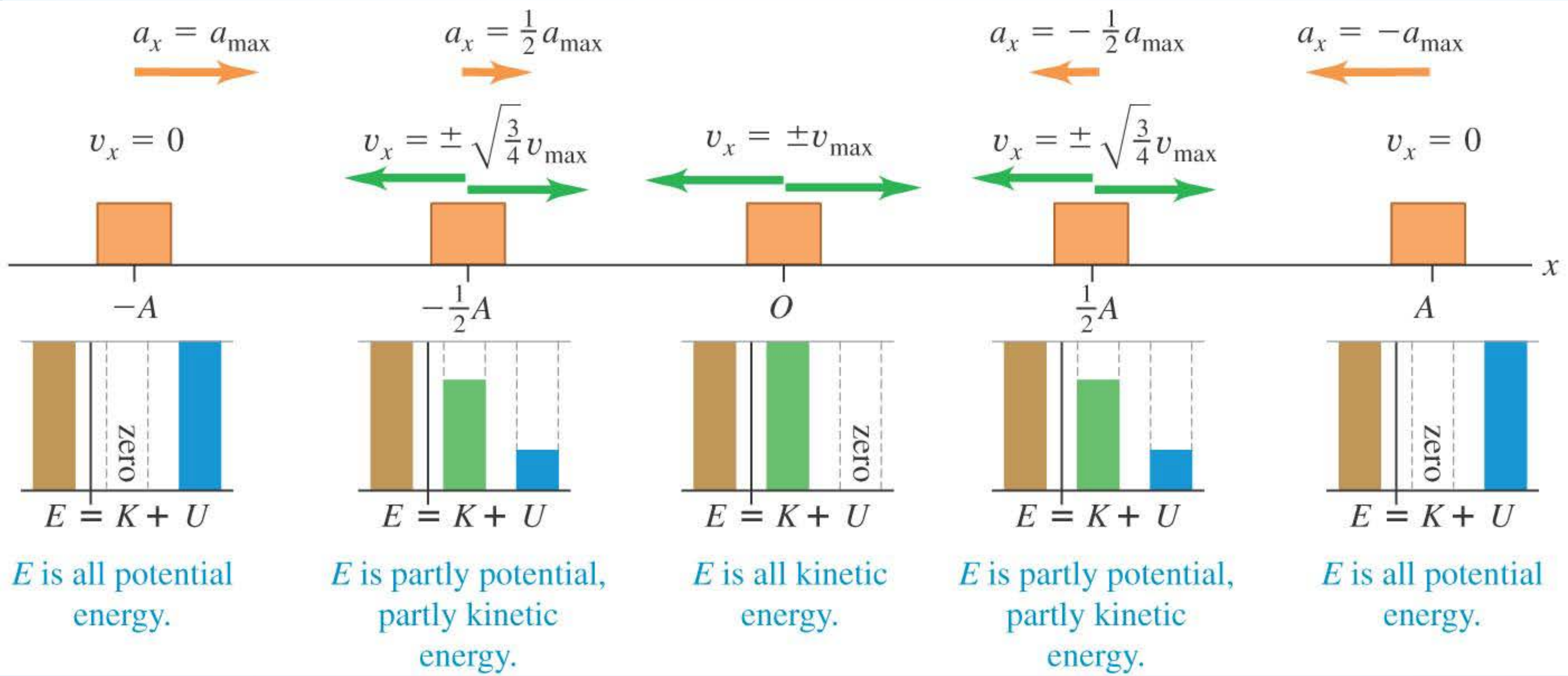
- Likewise, acceleration is the time-derivative of velocity (or the second derivative of displacement):

$$a_x(t) = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -\omega^2 A\cos(\omega t + \phi)$$

Energy in SHM

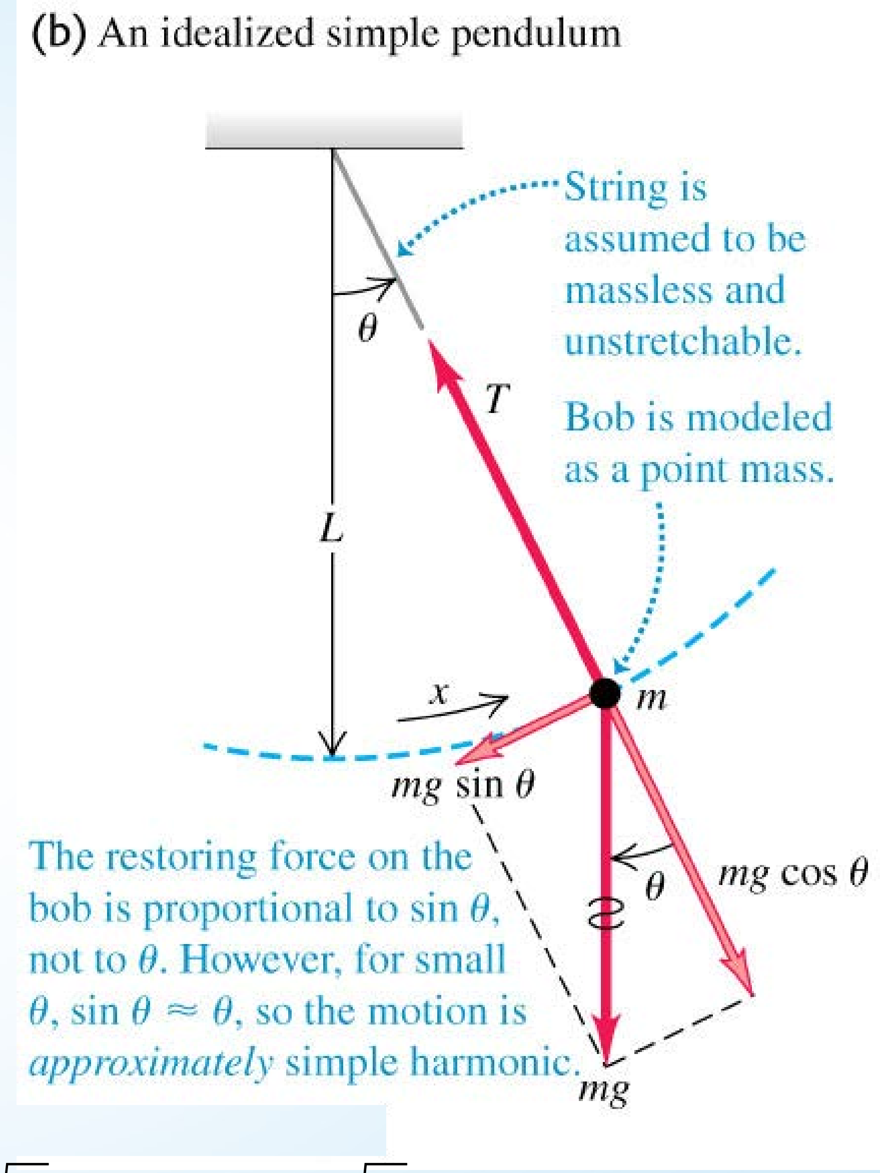
- The total mechanical energy $E = K + U$ is conserved in SHM:

$$E = \tfrac{1}{2} mv_x^2 + \tfrac{1}{2} kx^2 = \tfrac{1}{2} kA^2 = \text{constant}$$



The simple pendulum

- Other systems can show SHM.
- Consider a *simple pendulum* that consists of a point mass (the bob) suspended by a massless, unstretchable string (*physical pendulum*).
- If the pendulum swings with a small amplitude θ with the vertical, its motion is simple harmonic, where the restoring force is the component of gravity along the arc of the motion.



$$F(\theta) = ma_\theta = -mg\sin\theta$$

$$ml\frac{d^2\theta}{dt^2} \approx -mg\theta$$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\theta \qquad \omega = \sqrt{\frac{g}{l}}; \quad 2\pi f = \sqrt{\frac{g}{l}}; \quad T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$$

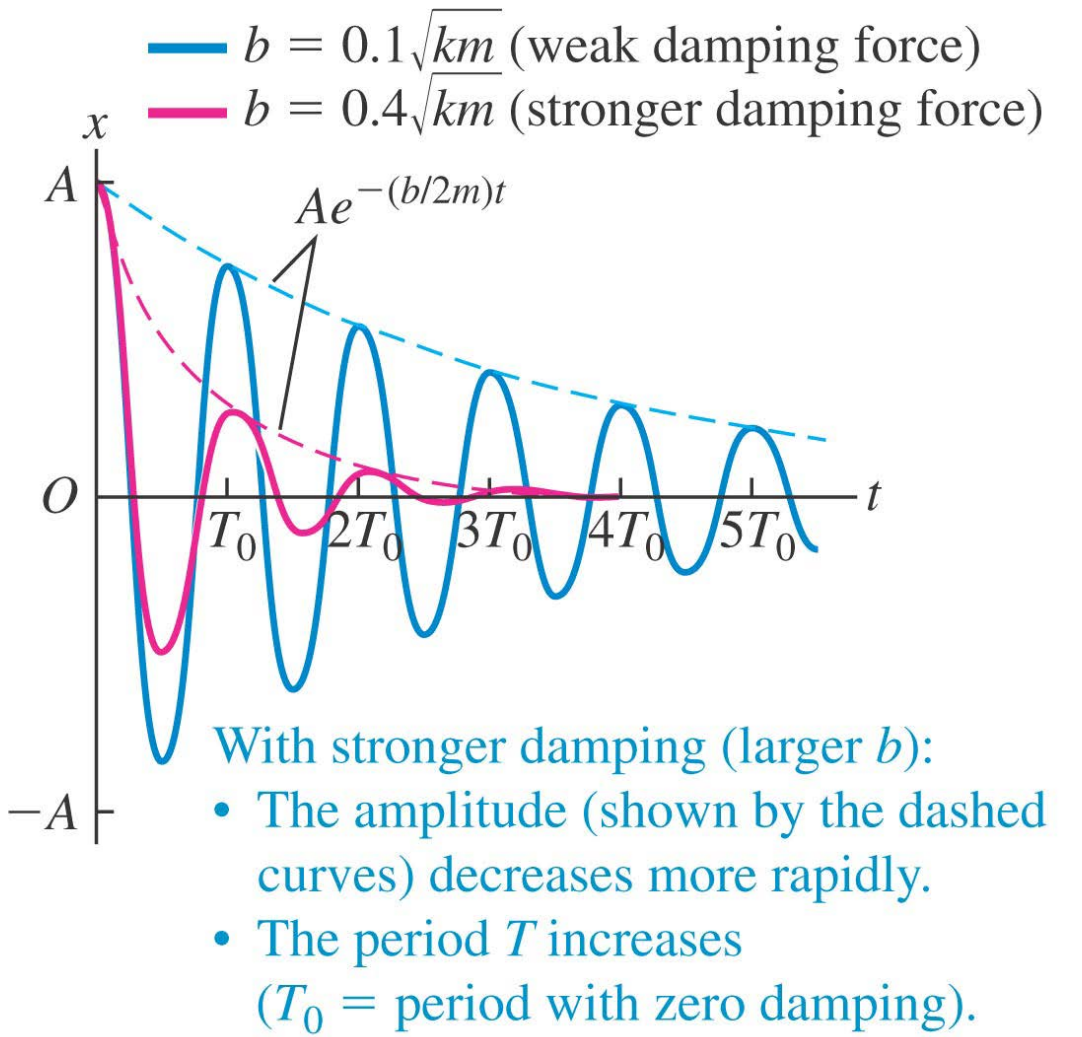
Damped oscillations

- Real-world systems have some dissipative forces that decrease the amplitude.
- Such dissipative forces are typically proportional to the speed *v*, and appear in the force equation with minus sign:

$$F_x = -kx - bv_x \quad \Rightarrow \quad m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

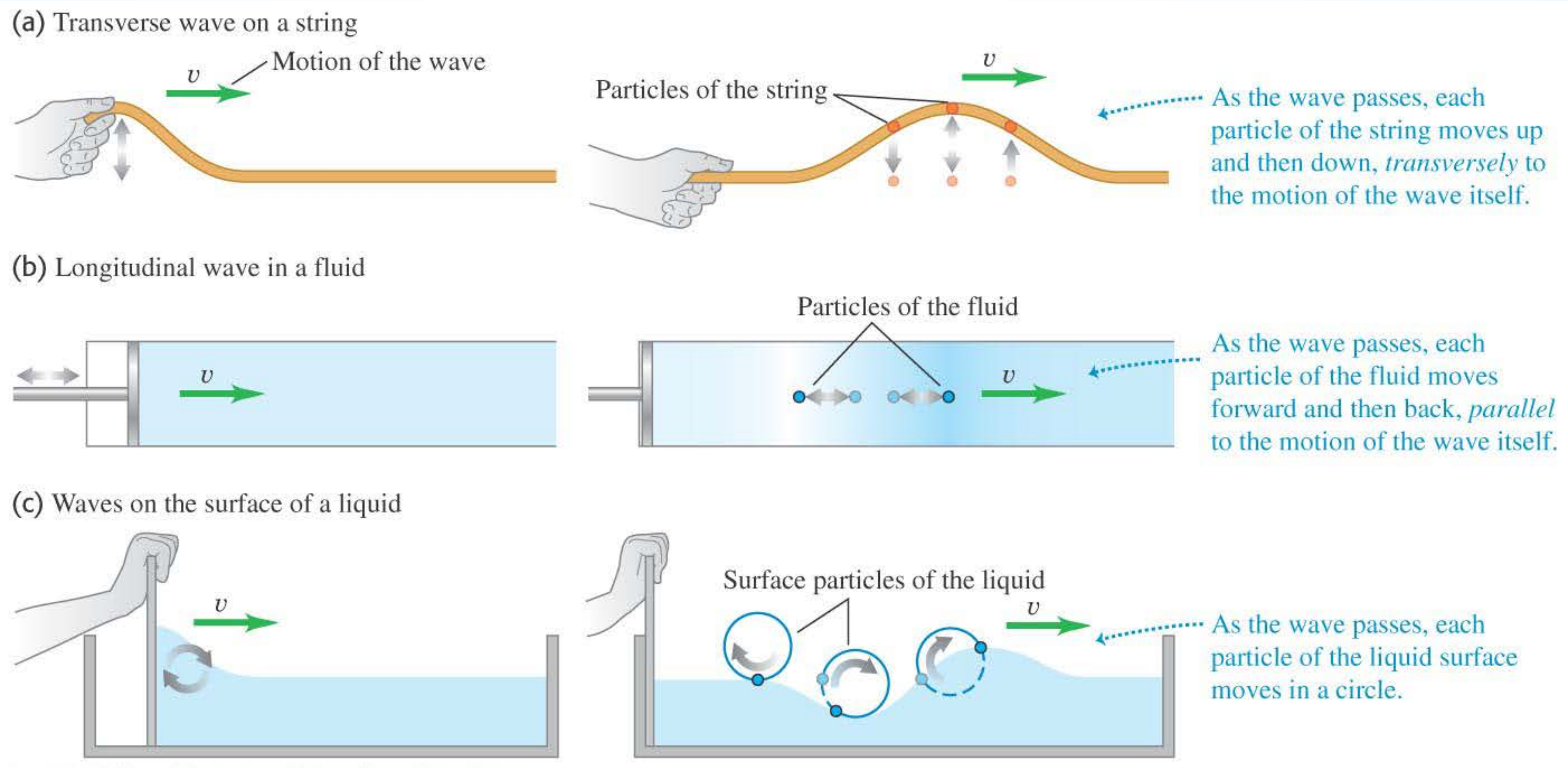
- The decrease in amplitude is called *damping* and the motion is called *damped oscillation*.
- The mechanical energy of a damped oscillator decreases continuously. The general solution is:

$$x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$$



Types of mechanical waves

- A *mechanical wave* is a disturbance traveling through elastic *medium*.
- Figure below illustrates *transverse waves* and *longitudinal waves*.

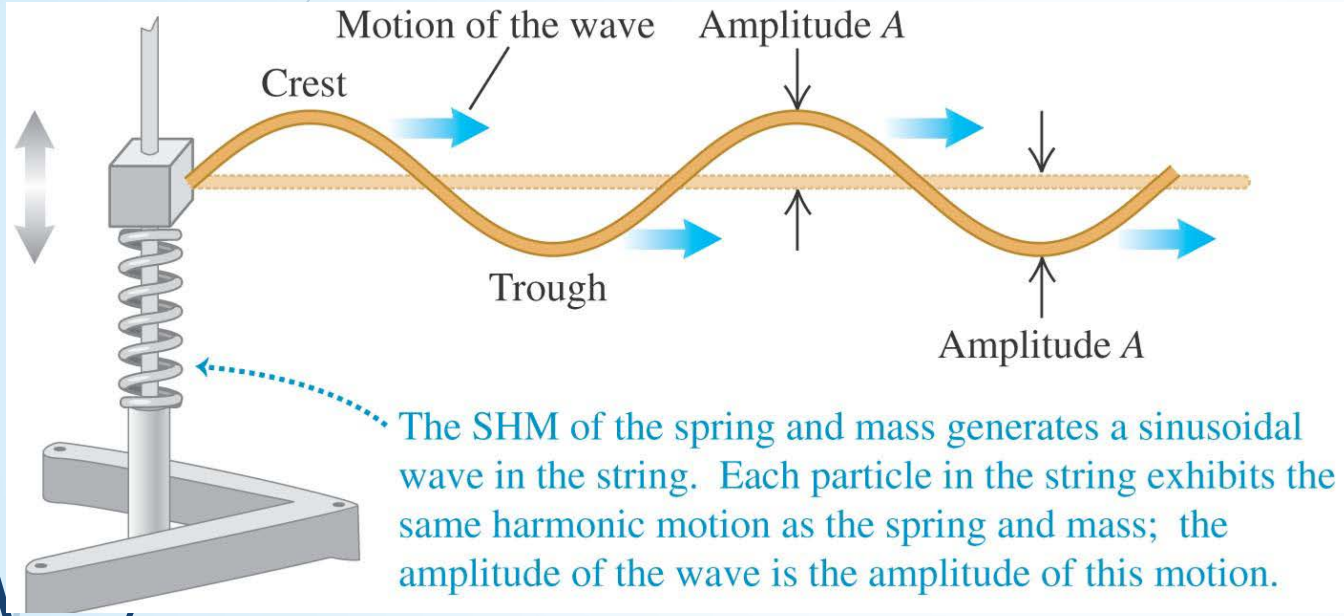


Periodic waves

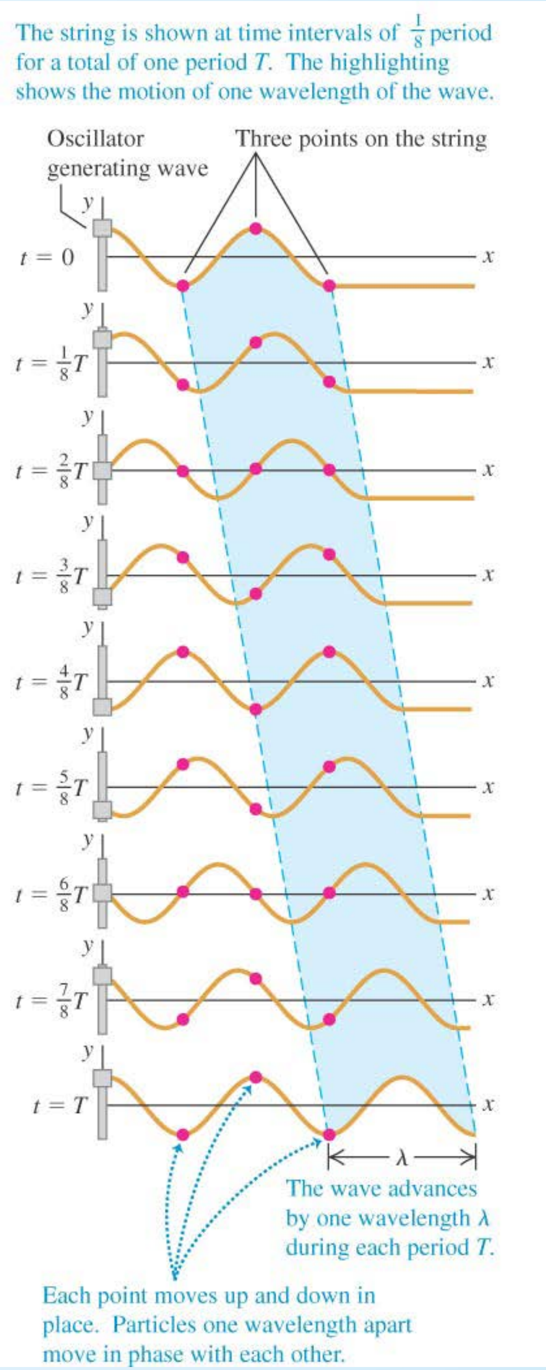
- For a *periodic wave*, each particle of the medium undergoes periodic motion. The speed of the wave is not the same as the speed of the particles.
- The *wavelength* λ of a periodic wave is the length of one complete wave pattern.
- The speed of any periodic wave of frequency f is $v = \lambda f$.
- Example: The speed of sound in air at 20° C is 344 m/s. What is the wavelength of a sound wave in air at 20° C if the frequency is 262 Hz?

Periodic transverse waves

- For the transverse waves shown here in Figures, the particles move up and down, but the wave moves to the right.

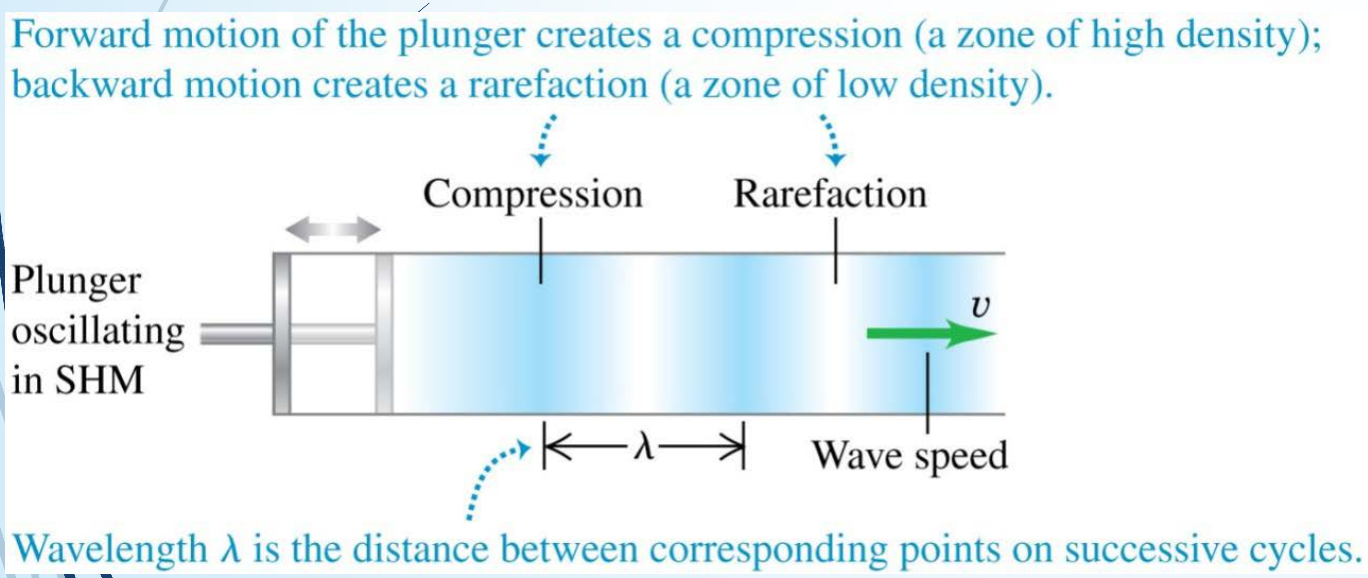


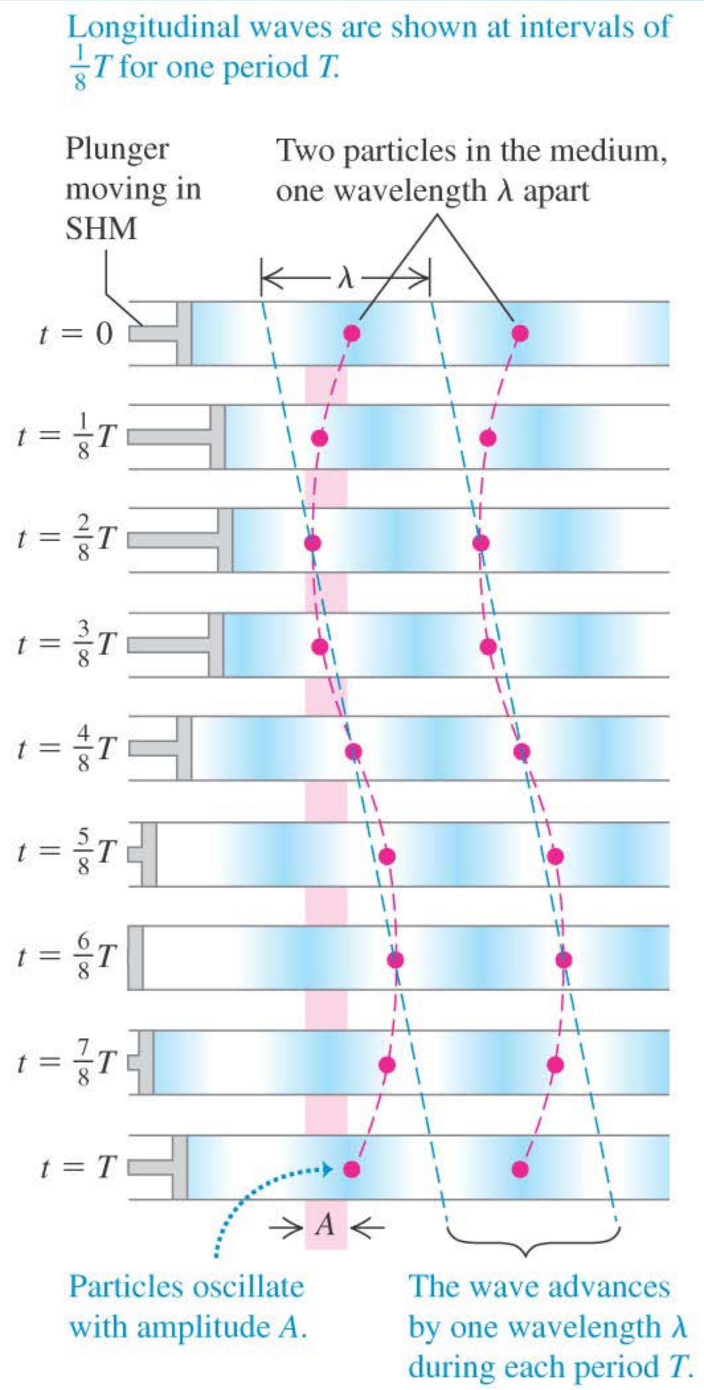
- This difference in direction of the waves and particles is why the wave is called a transverse wave.
- Note that the restoring force is transverse to the direction of the wave propagation.



Periodic longitudinal waves

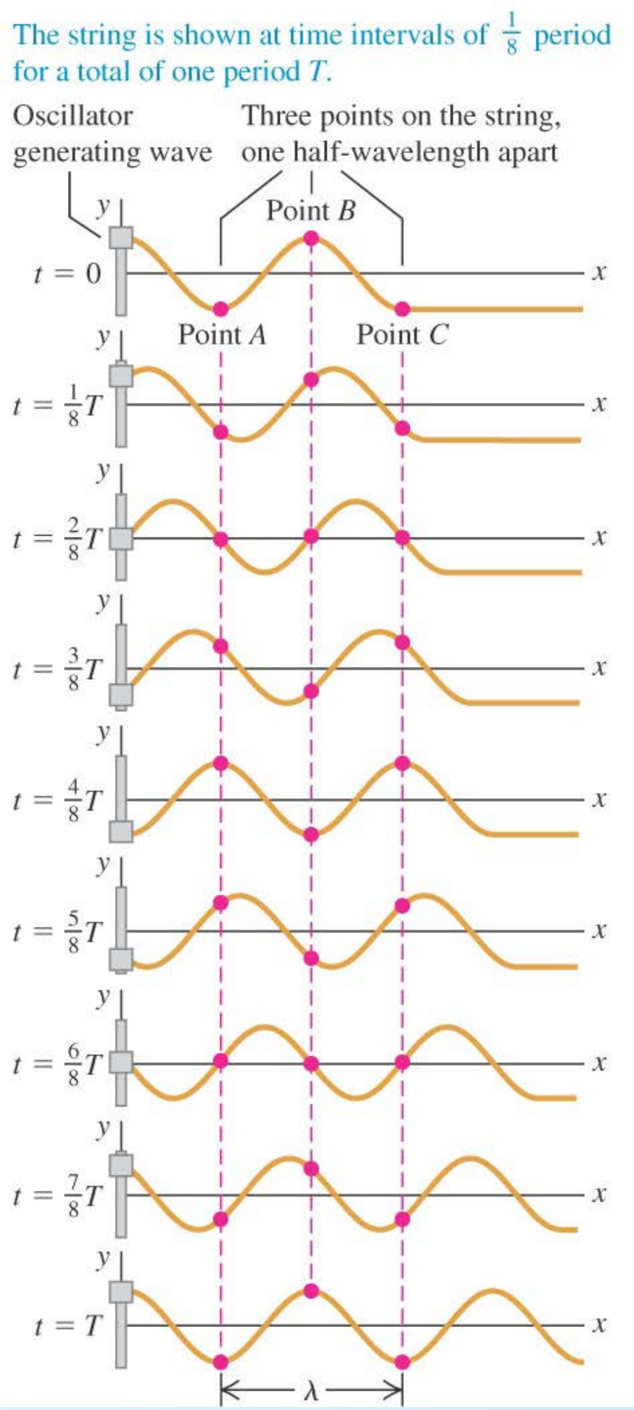
- For the longitudinal waves shown here in Figures, the particles oscillate back and forth along the same direction that the wave moves.
- The restoring force (pressure) is in the same direction as the wave propagation.





Mathematical description of a wave

- The *wave function*, $y(x,t)$, gives a mathematical description of a wave. In this function, y is the displacement of a particle at time t and position x.
- The wave function for a sinusoidal wave moving in the $+x$-direction is $y(x,t) = A\cos(kx - \omega t)$, where $k = 2\pi/\lambda$ is called the *wave number*.
- For transverse waves, y might represent the height of the wave at location x.
- For longitudinal waves, y might represent the pressure at location x.



Derivatives of *y*: wave equation

- Starting with $y(x,t) = A\cos(kx - \omega t)$, take *partial derivative* with respect to time to get *y* component of velocity:

$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$

- Likewise, take another partial derivative to get *y* component of acceleration:

$$a_y(x,t) = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

- We can also take partial derivatives with respect to *x* (instead of *t*) to get:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

- If we take the ratio of these two equations, we have:

$$\frac{\partial^2 y(x,t)/\partial t^2}{\partial^2 y(x,t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

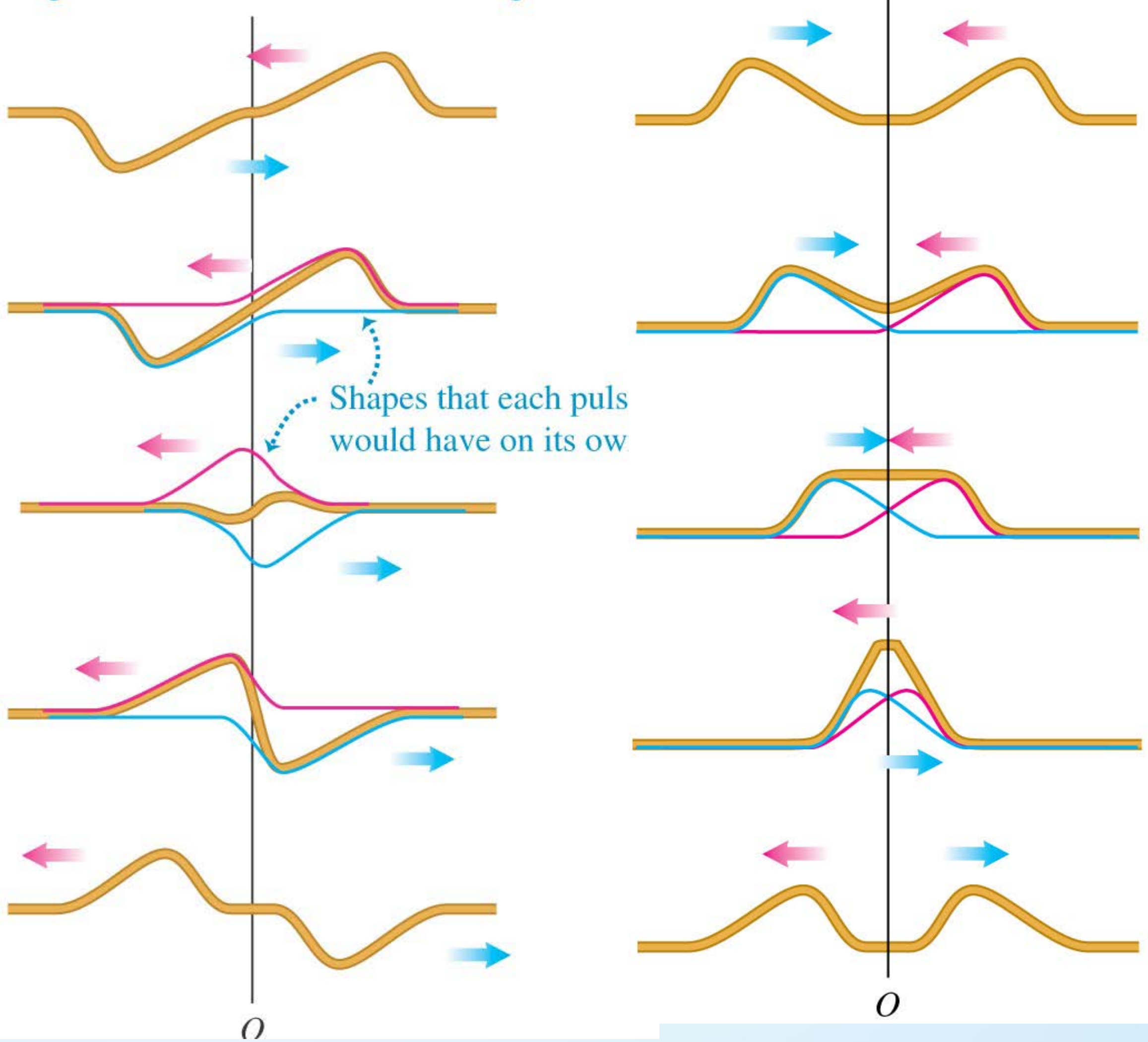
- Rearranging gives the wave equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Wave interference and superposition

- *Interference* is the result of overlapping waves.
- *Principle of super-position*: When two or more waves overlap, the total displacement is the sum of the displacements of the individual waves.
- Study Figures at the right.

As the pulses overlap, the displacement of the string at any point is the algebraic sum of the displacements due to the individual pulses.

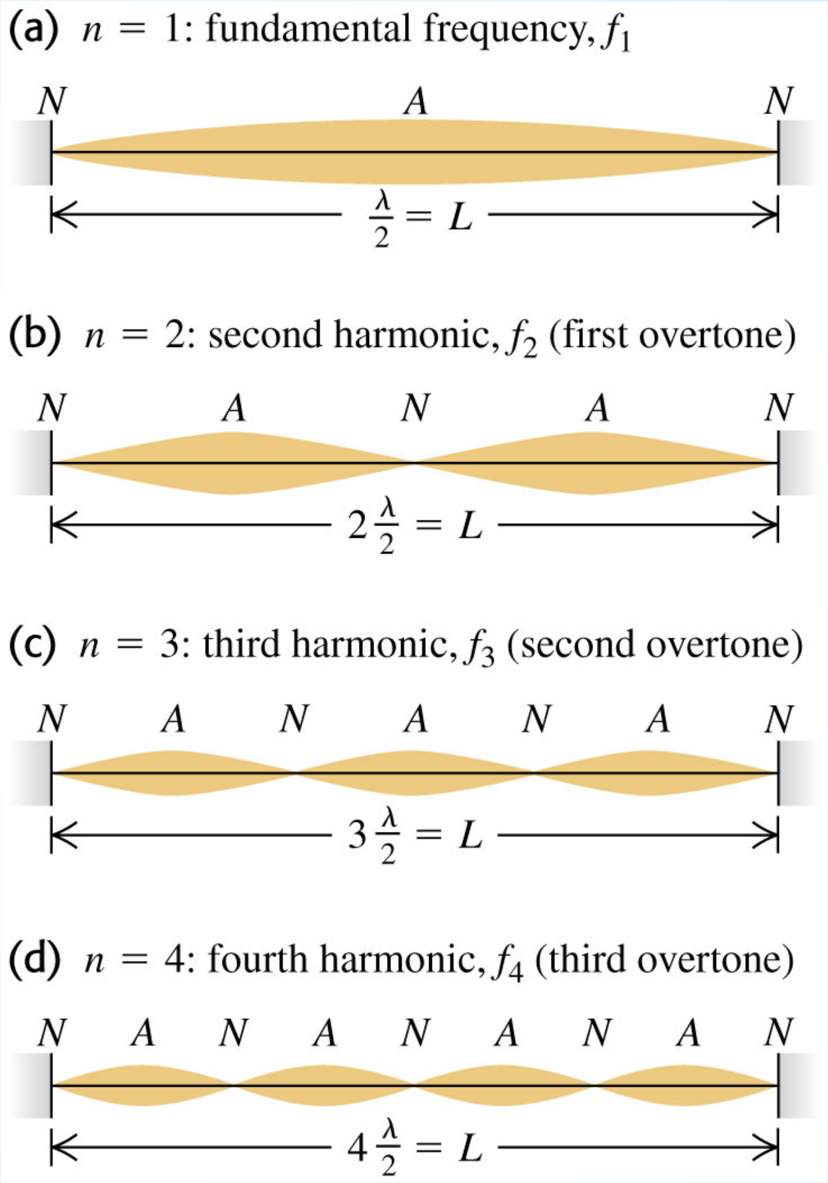


Normal modes of a string

- For a taut string fixed at both ends, the possible wavelengths are $\lambda_n = 2L/n$ and the possible frequencies are $f_n = n\,v/2L = nf_1$, where $n = 1, 2, 3, \ldots$

- f_1 is the *fundamental frequency*, f_2 is the second harmonic (first overtone), f_3 is the third harmonic (second overtone), etc.

- Figure illustrates the first four harmonics.



TYPES OF MUSCLES

- **Skeletal muscle**

  - striated muscle tissue existing under control of the somatic nervous system (voluntary control)

- **Cardiac muscle**

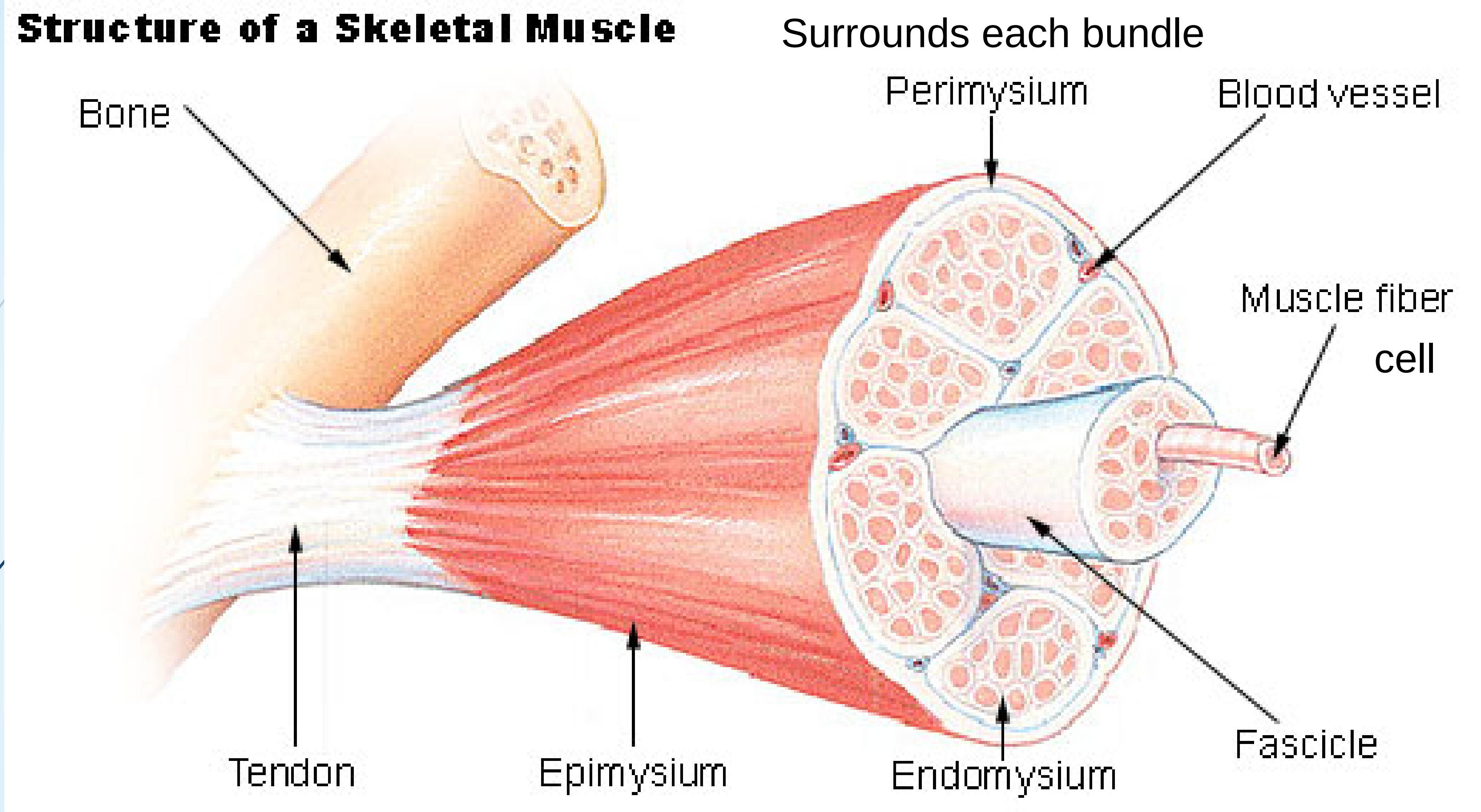
  - special striated muscle tissue of the heart working automatically and under the influence of autonomic nervous system (involuntarily)

- **Smooth muscle**

  - non-striated muscle tissue activated by autonomic nervous system, hormones, or simply stretching

Skeletal muscle

- attached to the bones for movement
- several **types of fibers** (speed, stamina, fatique, force, motor unit size, structure...)
- cells - long **multi-nucleated cylinders** - the length of muscle cell is a few mm (human sceletal muscle), the diameter is typically 100 - 150 µm
- **cytoskeleton** – supporting the cell shape



Bundles of collagen fibers

Fascia
Becomes the muscle sheath which fuses with the tendon

Surrounds each cell

Muscle fiber = muscle cell

- The **sarcolemma** – the cell membrane (plasma membrane) of a muscle cell

  - conduct stimuli

  - an outer coat (thin layer of polysaccharides with collagen fibrils) that fuses with a tendon fiber (they collect into bundles to form the muscle tendons)

- **Sarcoplasm** - cytoplasm with organelles
- **Myofibrils** - cylindrical organelles

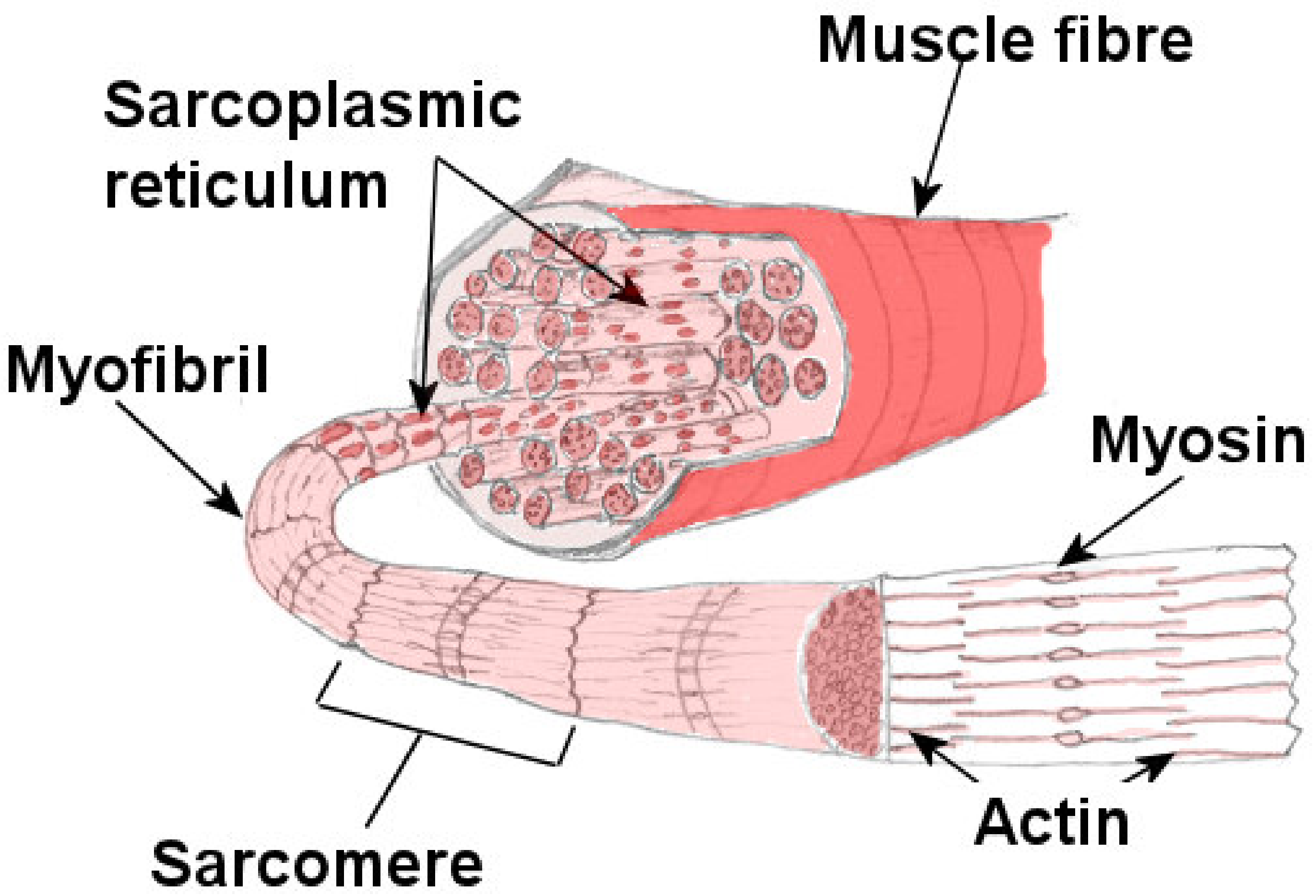
  – contractive elements the **actin** and **myosin** filaments (the length a few μm)

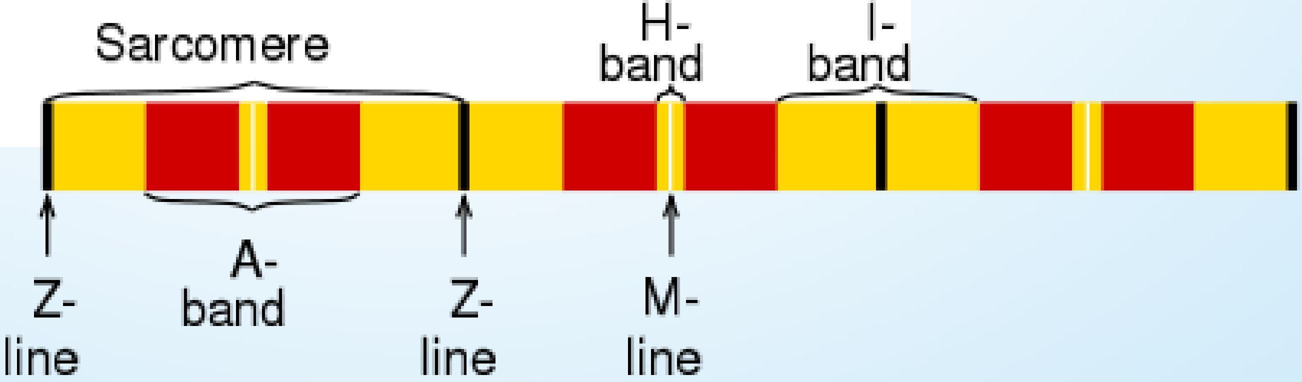
  – organized in repeated subunits along the length of the myofibril - **sarcomeres**

- **Sarcoplasmic reticulum** with **T- tubules** – internal

  conductive system

Structure of a muscle cell

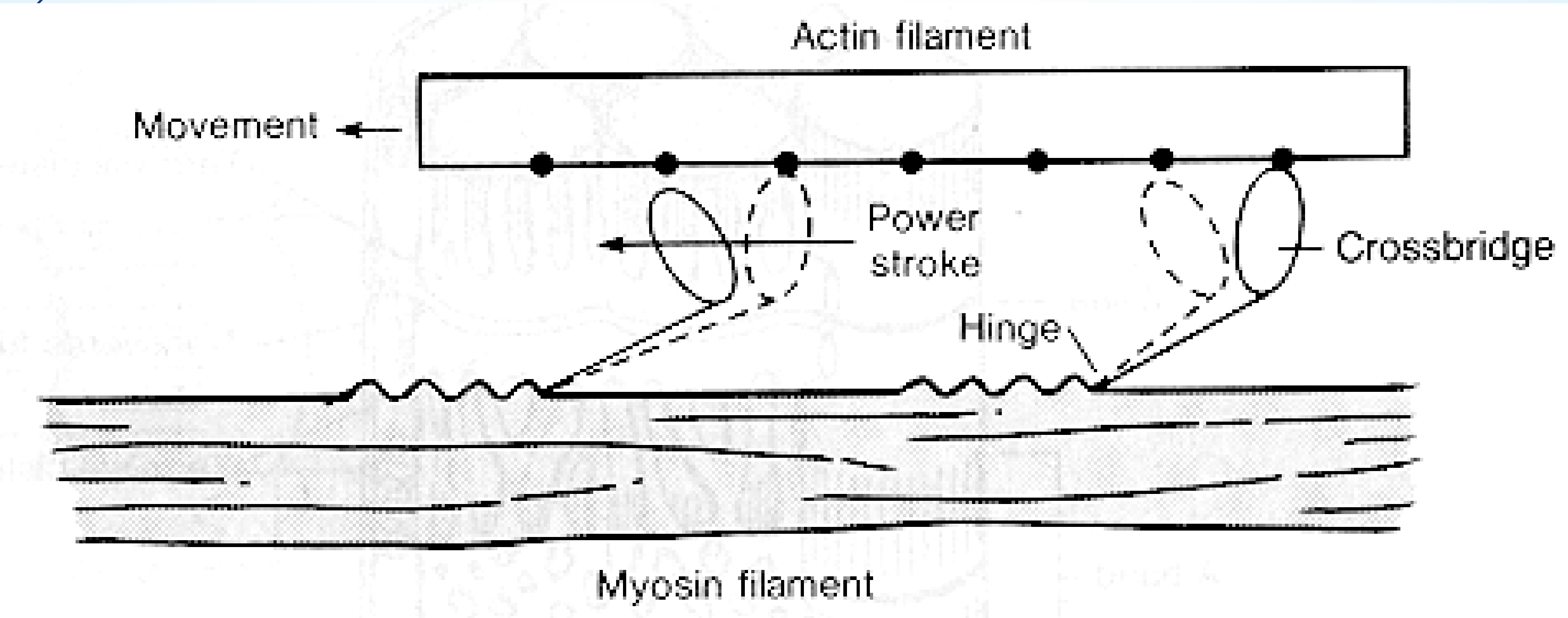




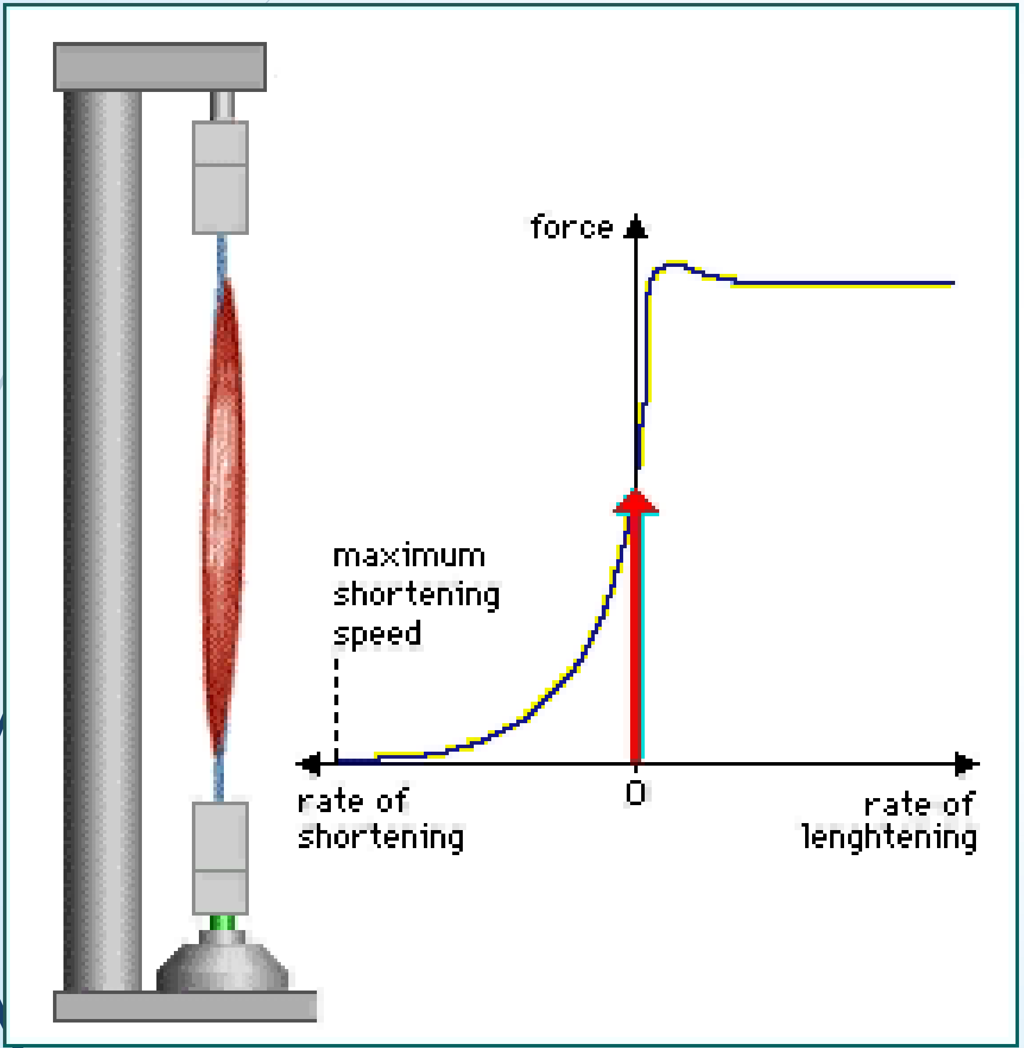
Muscle Contraction

We know that muscle shortening corresponds to the sliding of thin (actin) filaments past thick (myosin) filaments

Most widely accepted mechanism for generation of force is the formation of connections between these filaments by ***crossbridges***



Force-Velocity Relationship



Variables:

P	force generated by a muscle
V	speed of muscle contraction
a, b, c	constants
f1	rate constant of crossbridge attachment
g1	rate constant of crossbridge detachment
Pmax	maximum force

Hill Equation

Hill (1938) hypothesized specific relationships between the force generated by a muscle and the speed at which a stimulated muscle contracts under a given load. A stimulated muscle may contract to 1/3 its size at a particular speed. When that muscle is attached to a load, the speed and size to which it contracts decreases. In other words, as the load increases, the muscle cannot lift the load as far. Hill expressed this, for each sarcomere in a muscle, as the characteristic equation

$$(P + a)(V + b) = c$$

where P describes the force generated by a muscle, V is the speed at which a muscle contracts, and a, b, c are constants. The constant a describes the force expended to make the muscle contract, and b describes the smallest contraction rate of the muscle.

We can interpret Hill's equation in the following way: as the force (P) being exerted by the muscle increases, the contraction rate (V) must decrease so that we maintain the constant, c. You can see that this makes sense by facing a friend and placing your palm flat against his. Have your friend offer resistence as you push his hand away. When your friend offers little resistance, you can rapidly displace his hand (high V) with little force (low P). When your friend offers maximal resistance, you will need maximum force (high P) to slowly (low V) displace his hand.

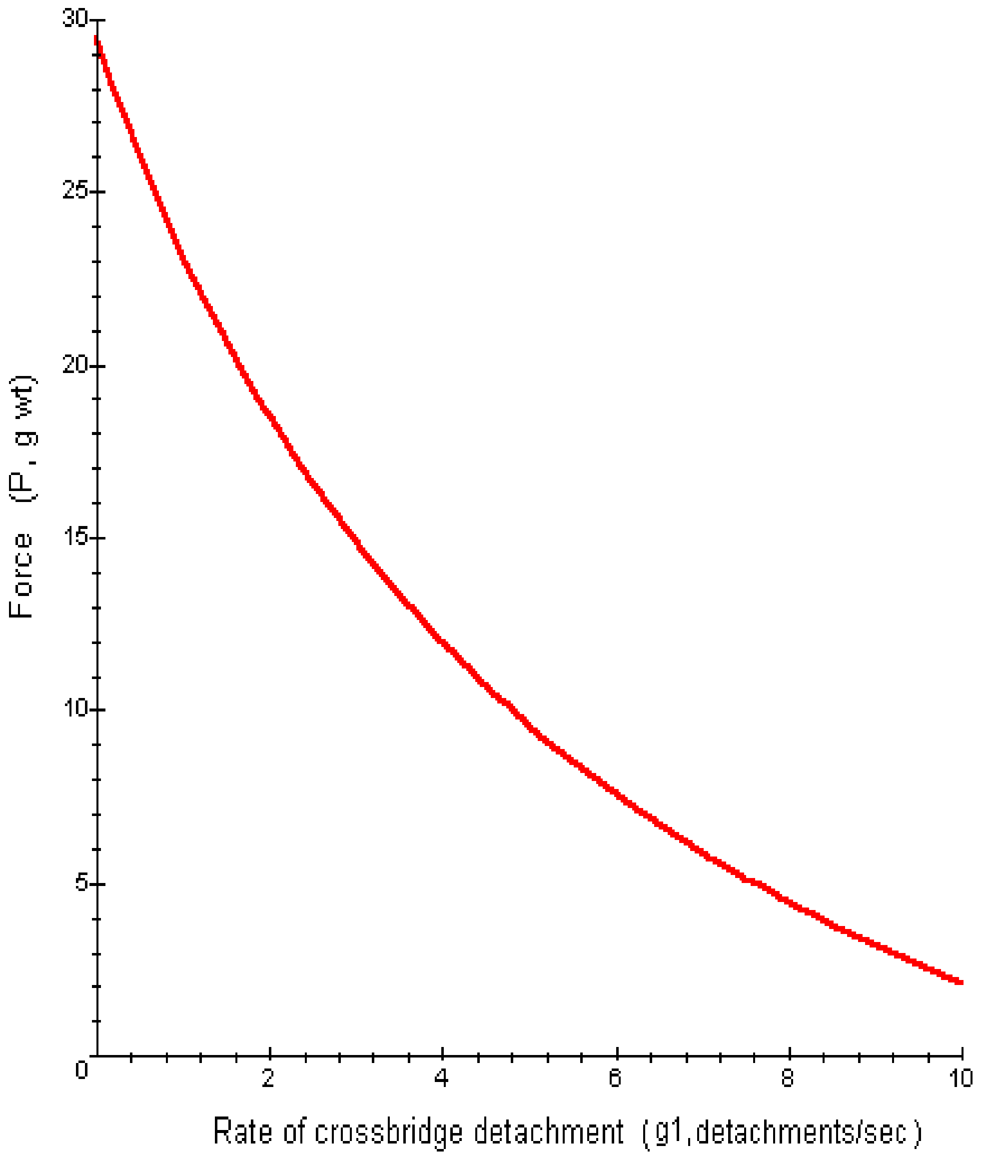
The force (P) is now a function of the speed of contraction (V, cm/sec) of a stimulated muscle. Hill estimated that ab = g1/f1 where f1 is the rate constant (attachments per second) for crossbridge connection as actin and myosin molecules combine to form crossbridges. The rate constant g1 is the rate (detachments per second) for crossbridge detachment. The constant a is approximately equal to 1/4 Pmax. Pmax is the force generated in the sarcomere if all actin sites were attached to myosin sites.

$$P = \frac{c\, f_1 P_{max}}{f_1 V P_{max} + 4 g_1} - \frac{1}{4} P_{max}$$

We can plot g1 versus P to predict how the force generated by a muscle depends on the rate at which crossbridges detach. From Hill's data we choose V as 2 cm/sec, f1 as 0.21 attachments/sec, Pmax as 57.4 g. wt., and c as 87.6 g wt cm/sec.

Notice in the graph, the force generated by a muscle is high when the rate of crossbridge detachment is low. When the rate of detachment is low, the muscle is highly contracted and there is a great deal of overlap between thick and thin filaments. As the rate of detachment increases, the degree of overlap decreases, as does the force generated by the muscle.

Huxley later derived a more precise equation based on the same principles relating force to the rate of crossbridge detachment.

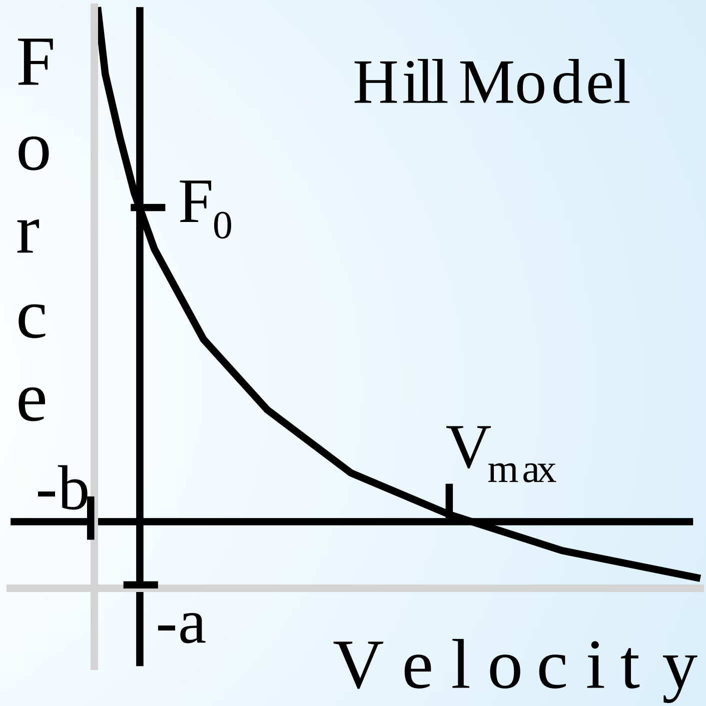


Hill's relationship can be rewritten as

- The Hill equation describes shortening muscle:

$$(P + a)v = b(P_{max} - P)$$

- Here, a and b are constants,
  P_{max} is maximum force,
  P is force, and
  v is velocity.



Hill also suggested that the mechanics of muscle contraction is closely linked to the muscle’s energy metabolism, because in his experiments the same hyperbolic force–velocity relationship could be derived from heat measurements, and the constant *a* was found to match closely to an empirically derived thermal constant of shortening heat, α. However, α was later found not to be a constant but dependent on shortening velocity and load. It appears, therefore, that the force–velocity behavior of a muscle is not an unfiltered manifestation of energetic events occurring inside the muscle, as Hill originally thought.

Control Questions

1. Examples of periodic motion.
2. Characteristics of SHM.
3. Periodic transverse waves.
4. Periodic longitudinal waves.
5. Normal modes of a string.
6. Structure of a skeletal muscle.
7. Structure of a muscle cell. Contraction.
8. Hill equation.
9. Heat production.
10. Efficiency.

Recommended literature:

Basic:

1. Vladimir Timanyuk, Elena Zhivotova, Igor Storozhenko. Biophysics: Textbook for students of higher schools / Kh.: NUPh, Golden Pages, 2011.- 576p.
2. Vladimir Timaniuk, Marina Kaydash, Ella Romodanova. Physical methods of analysis / Manual for students of higher schools/– Kharkiv: NUPh: Golden Pages, 2012. – 192 p.
3. Philip Nelson. Biological Physics. – W. H. Freeman, 1st Edition, 2007. – 600 p.
4. Biophysics, physical methods of analysis. Workbook: Study guide for the students of higher pharmaceutical educational institutions / Pogorelov S. V., Krasovskyi I. V., Kaydash M. V., Sheykina N. V., Frolova N. O., Timaniuk V. O., Romodanova E.O., Kokodii M.H. – Kharkiv., – 2018. – 130 p.
5. Center for distance learning technologies of NPhaU. Access mode: http://nuph.edu.ua/centr-distancijjnih-tehnologijj-navcha/

Support:

1. Eduard Lychkovsky. Physical methods of analysis and metrology: tutorial / Eduard Lychkovsky, Zoryana Fedorovych. – Lviv, 2012. – 107 p.
2. Daniel Goldfarb. Biophysics DeMYSTiFied. – McGraw-Hill Professional, 1st Edition, 2010. – 400 p.

Thanks for your attention