TOPIC 7.A. Magnetostatics. The Motion of Charged Particles in the Electric Field. Electromagnetic Oscillations and Waves

Magnetostatics

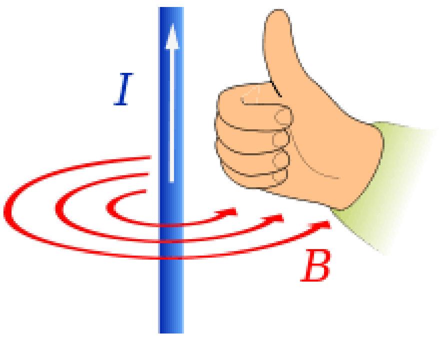
Exercise 7.1a. Give definitions of terms, define the parameters of formulas (7.1a) – (7.7a) and their units of measurement:

Magnetic field is ____

____

The power characteristic of the magnetic field at each point of space is the *magnetic induction vector* $\vec{B}$ [T].

The magnetic moment of the current-carrying loop is:



$$p_m = IA, \tag{7.1a}$$

p_m is ____

I is ____

A is ____

Magnetic flux: $$\Phi = BA\cos\alpha, \tag{7.2a}$$

Φ is ____

B is ____

α is ____

Magnetic field induction: $$\vec{B} = \mu_0\mu\vec{H}, \tag{7.3a}$$

H is ____

μ is ____

μ_0 is ____

The Ampere`s force ____

____

$$F = IBl\sin\alpha. \tag{7.4a}$$

The Lorentz force ____

____:

$$\vec{F} = q\left[\vec{\upsilon}\times\vec{B}\right]. \tag{7.5a}$$

The magnetic flux density in the center of the loop with current is equal to:

$$B = \frac{\mu\mu_0 I}{2R} \tag{7.6a}$$

R is ____

The magnetic flux density inside a long solenoid on its axis is equal to:

$$B = \mu\mu_0 I\frac{N}{l} = \mu\mu_0 In, \tag{7.7a}$$

N is ____

l is ____

n is ____

Problem 7.2a. Induction of the magnetic field in the center of the circular contour is $B = 20$ μT. Calculate the magnetic moment of the circuit if the current in it is $I = 1$ A.

Data:	***Solution:***

Answer: ____

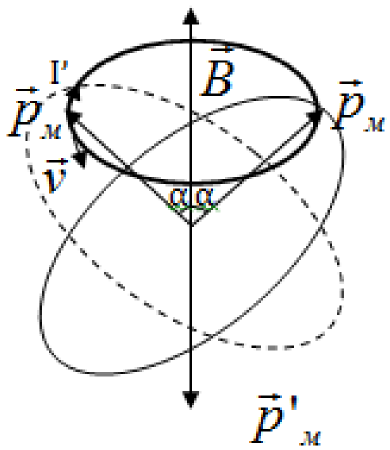
Magnetic Properties of Objects

Exercise 7.3a. Complete the Table 7.1a, define the parameters and their units of measurement.

Table 7.1a

Basic parameters and their units of measurement

Title	Formula
Electron magnetic moment	(7.7a)
Orbital magnetic moment	(7.8a)
Gyromagnetic ratio	(7.9a)
Spin gyromagnetic ratio	(7.10a)
Bohr magneton	(7.11a)



P is ______________________________

r is ______________________________

e is ______________________________

v is ______________________________

m_e is ______________________________

υ is ______________________________

μ_B is ______________________________

h is ______________________________

Electromagnetic Induction

Exercise 7.4a. Define the following terms, define the parameters and their units of measurement:

Electromotive force of induction $\mathcal{E}_i$ is ______________________________

____

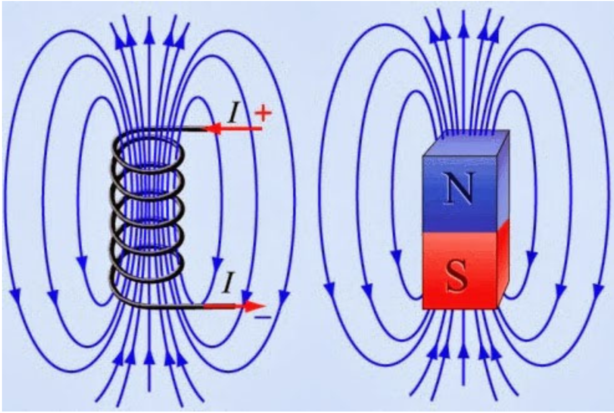
Faraday's-Neumann law:

$$\mathcal{E}_i = -\frac{d\Phi}{dt} = -L\frac{dI}{dt} \qquad (7.12a)$$

N is ____

Magnetic flux through a solenoid is equal to:

$$\Phi = NBS = N\mu_0\mu I\frac{N}{l}A = \mu_0\mu n^2 IV\,, \qquad (7.13a)$$



where Φ is ______________________________

N is ______________________________

A is ______________________________

B is ______________________________

n is ______________________________

V is ______________________________

l is ______________________________

The magnetic field energy of the current I:

$$W_H = \frac{LI^2}{2} = \frac{\mu_0\mu H^2}{2}V\,, \qquad (7.14a)$$

where W_H is ______________________________

L is ______________________________

I is ______________________________

H is ______________________________

V is ______________________________

Alternating Current

Alternating current (AC) is a current which strength varies in time.

The alternating current is determined by *Ohm's law*:

$$I = \frac{U}{R} = \frac{U_0}{R}\cos\omega t = I_0\cos\omega t\,, \qquad (7.15a)$$

where I_0 is amplitude value of current.

Inductive reactance:

$$X_L = \omega L\,. \qquad (7.16a)$$

Capacitive reactance:

$$X_c = \frac{1}{\omega C}. \qquad (7.17a)$$

Total *impedance* of the AC circuit:

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}\,. \qquad (7.18a)$$

Total *reactance* of the AC circuit:

$$X = X_L - X_C = \omega L - \frac{1}{\omega C}. \qquad (7.19a)$$

Resonance frequency: $$\omega_{res} = \frac{1}{\sqrt{LC}}. \qquad (7.20a)$$

Exercise 7.5a. Describe the parameters from the previous formulas, specify their units of measurement.

R is ______________________________

ω is ______________________________

t is ____________________

L is ____________________

C is ____________________

φ is ____________________

Problem 7.6a. Find the strength and power of an alternating electric current ($f = 50 Hz$),passed through the human body, if he touches with his hands to the network cables with voltage $U = 220\ V$. Let the resistance for the person`s body is $R = 10^3\ \Omega$ and capacity is $C = 5 \cdot 10^{-8}\ F$.

Data:	***Solution:***

Answer: ____________________

Electromagnetic Oscillations and Waves

Exercise 7.7a. Define the following terms, the parameters and their units of measurement:

Electromagnetic oscillations are ____________________

____

Differential equations that describe different types of electromagnetic oscillations are:

- *Free undamped oscillations*:

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0 \qquad (7.21a)$$

- *Damped oscillations*:

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = 0 \qquad (7.22a)$$

- *Forced fluctuations*:

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = \frac{U_0}{L}\cos\omega t \qquad (7.23a)$$

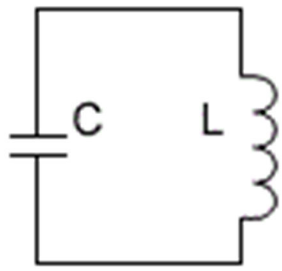


Fig. 7.1a. The ideal oscillating contour

q is ____________________

t is ____________________

ω_0 is ____________________

β is ____________________

L is ____________________

U_0 is ____________________

The period of oscillations is determined by the *Thomson formula*:

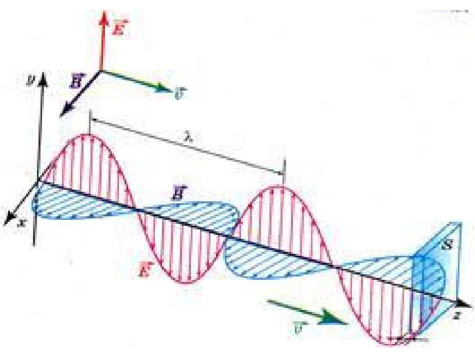
$$T = 2\pi\sqrt{LC}. \qquad (7.24a)$$

Damping of oscillations is characterized by the *damping logarithmic decrement:*

$$\lambda = \ln\frac{A(t)}{A(t+T)} = \beta T . \qquad (7.25a)$$

where *A (t)* is the amplitude of the corresponding quantity.

Exercise 7.8a. Define the following terms, the parameters and their units of measurement:



Electromagnetic wave is ______________________________

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The light propagation speed in a medium is equal to:

$$\upsilon = \frac{c}{\sqrt{\varepsilon\mu}} = \frac{c}{n} \qquad (7.26a)$$

The absolute refractive index of the medium n is equal to:

$$n = \sqrt{\varepsilon\mu} , \qquad (7.27a)$$

where c is ______________________________________

ε is ____

μ is ____

Example of problem solution.

The oscillatory circuit contains a capacitor with a capacity of 800 pF and an inductor of 2 mkH. Calculate the circuit natural oscillation period.

Data:	***Solution:***
C =800 pF $=8\times10^{-10}$ F L = 2 mkH $= 2\times10^{-6}$H	Using the Thomson formula: $T = 2\pi\sqrt{LC}$ $T = 2\pi\sqrt{LC} = 2\pi\sqrt{2\times10^{-6}\times8\times10^{-10}} =$ $= 0.25\times10^{-6} = 0.25\,\mu s$
T - ?	

Answer: the circuit natural oscillation period is 0.25 μs.

Problem 7.9a. The speed of electromagnetic waves propagation in a certain medium is $\upsilon = 250\ \text{mm}\cdot\text{s}^{-1}$. Determine the wavelength of electromagnetic waves in this medium, if their frequency in vacuum is $\nu = 1$ MHz.

Data:	***Solution:***

Answer: ____

Control questions

1. How do magnetic fields affect living organisms?
2. List the sources of magnetic fields in the living body.
3. What biological processes occur in the body tissues under the influence of AC?

4. Types of electromagnetic oscillations.
5. The main characteristics of electromagnetic waves.
6. Types of electromagnetic radiation.
7. Electrophoresis. It`s applying in medicine.
8. Diode dynamics and physiotherapy. It`s applying in medicine.
9. Application of electro-magnetic fields in cosmetology.

Individual assignments

1. Determine the active resistance of the electromagnetic relay coil in the circuit of the X-ray machine, if the inductance of the coil is 150 H, the current in it is 2.5 mA, the voltage is 120 V and the frequency is 50 Hz.
2. Calculate the length of half-wave dipole which is radiates electromagnetic waves into space, if a microwave device with a frequency of 40.68 MHz is used as a generator.
3. The oscillatory circuit of the apparatus for therapeutic diathermy consists of an inductor and a capacitor with a capacity of 300 pF. Determine the coil inductance if the generator frequency is 1 MHz.
4. In the oscillatory circuit, for a general D'arsonvalization, a large solenoid has the inductance of L = 125 μH, and a capacitance of C = 5 μF. Find the frequency of electromagnetic oscillations.
5. Calculate the inductance of the microwave device oscillatory circuit, if it`s capacitance is C = 2 pF and oscillations frequency is ν = 40 MHz.
6. Including that the circuit resonates at the wavelength λ = 630 m, determine the relative permittivity of the medium filling the space between the plates of the capacitor.
7. The oscillating circuit contains a solenoid (length l = 5 cm, cross-sectional area A_1 = 1.5 cm^2, number of turns N = 500) and plane condenser (distance between plates is d = 1.5 mm, plate area is A_2 = 100 cm^2). Determine the frequency ω of the natural oscillations.
8. Determine the logarithmic decrement, at which oscillatory circuit energy decreases in n = 8 times during N = 5 total oscillations.

TOPIC 7.B. Bioelectric Potentials

The Resting Membrane Potential and the Action Potential

Exercise 7.1b. Define the following terms and answer the questions:

Nerve impulse is ____

____

Resting potential is ____

____

Action potential is ____

____

Axon is ____

____

*The gradient of the concentrations of which ions does cause the resting potential?*__________

____

If the membrane is permeable to certain ions and impermeable to others, then the equilibrium potential will be described by the *Nernst equation:*

$$\Delta\varphi = \frac{RT}{zF} ln\frac{c_o}{c_i} \qquad (7.1b)$$

Exercise 7.2b. Write down the definitions of the parameters, their measurement units and the numerical values of the constants in the Nernst equation:

$\Delta\varphi$ is ____

R is ____

T is ____

z is ____

F is ____

c_o is ____

c_i is ____

Example of problem solution.

The equilibrium potassium potential if the potassium concentration inside the cell is c_i = 10 mmol·L^{-1}, outside is c_o = 500 mmol·L^{-1} and temperature is t = 27 °C, will be calculated by the following way:

Data:	***Solution:***
c_i = 10 mmol·L^{-1} c_o = 500 mmol·L^{-1} t = 27 °C=300 K K^+ $\Delta\varphi$ – ?	$\Delta\phi = \frac{8.31\times 300}{(+1)\times 9.65\times 10^4}\ln(\frac{10\times 10^{-3}}{500\times 10^{-3}}) = \frac{2493}{9.65\times 10^4}\ln(0.02) =$ $= 258.34\times 10^{-4}\times(-3.9) = -1007.54\times 10^{-4}(V) \approx -101(mV)$

Answer: *$\Delta\varphi$= –100 mV*

The membrane potential is calculated by the *Goldman-Hodgkin-Katz equation:*

$$\Delta\varphi = \frac{RT}{F}\ln\frac{P_K\cdot[K^+]_o + P_{Na}\cdot[Na^+]_o + P_{Cl}\cdot[Cl^-]_i}{P_K\cdot[K^+]_i + P_{Na}\cdot[Na^+]_i + P_{Cl}\cdot[Cl^-]_o}, \qquad (7.2b)$$

where P_K, P_{Na}, P_{Cl} are the membrane permeability constants for K^+, Na^+, Cl^- respectively;

$[K^+]_o$, $[Na^+]_o$, $[Cl^-]_o$ are the ions concentrations outside the cell ($mol \cdot m^{-3}$);

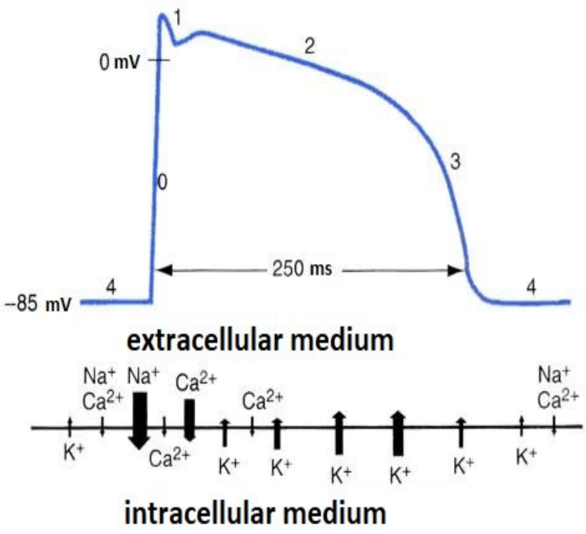
$[K^+]_i$, $[Na^+]_i$, $[Cl^-]_i$ are the ions concentrations inside the cell ($mol \cdot m^{-3}$).

At resting state, the permeability constants for ions K^+, Na^+ and Cl^- are related as:

$P_K : P_{Na} : P_{Cl} = 1 : 0.04 : 0.45$ (7.3b)

In the excitation state, the permeability constants of the membrane are related as:

$P_K : P_{Na} = 1 : 20$ (7.4b)



Problem 7.3b. Calculate the cell temperature if the internal concentration of chloride ions is $c_i = 9$ mmol·L^{-1}, external concentration is $c_o = 125$ mmol·L^{-1}, resting potential is$\Delta\varphi_{rp} = -70$ mV.

Data:	***Solution:***

Answer:____

Problem 7.4b. Calculate, how many times the concentration of potassium ions inside the cell should exceed the concentration from the outside in order for the rest potential to be $\Delta\varphi_{rp} = -91$ mV in conditions that the temperature is 37 °C.

Data:	***Solution:***

Answer:____

Example of problem solution.

Determine the value of the action potential $\Delta\varphi_{AP}$ inside the frog muscle fiber cell if the concentration of sodium ions inside is 1 mmol/L and outside 7 mmol/L, concentration of potassium ions inside is 48 mmol/L and outside 1 mmol/L. The temperature of the tissue is 27 °C.

Data:

$$[K^+]_i = 48 \text{ mmol/L}$$
$$[K^+]_o = 1 \text{ mmol/L}$$
$$[Na^+]_i = 1 \text{ mmol/L}$$
$$[Na^+]_o = 7 \text{ mmol/L}$$
$$T = 27 + 273 = 300\ K$$

$\Delta\varphi_{AP}$=?

Solution:

The value of the membrane potential calculated according to the equation Goldman–Hodgkin–Katz:

$$\Delta\varphi = \frac{RT}{F}\ln\frac{P_K\cdot[K^+]_o + P_{Na}\cdot[Na^+]_o + P_{Cl}\cdot[Cl^-]_i}{P_K\cdot[K^+]_i + P_{Na}\cdot[Na^+]_i + P_{Cl}\cdot[Cl^-]_o}$$

Where the P_K, P_{Na}, P_{Cl} are membrane permeability constants for K +, Na +, Cl-, respectively.

To calculate the action potential permeability constants we should take the ratio $P_K : P_{Na} = 1 : 20$.

So:

$$\Delta\varphi_{AP} = \frac{RT}{F} ln \frac{P_K\cdot[K^+]_o + P_{Na}\cdot[Na^+]_o}{P_K\cdot[K^+]_i + P_{Na}\cdot[Na^+]_i} =$$
$$= \frac{8.31\cdot 300}{96.5\times 10^3}\ln(\frac{1\cdot 1 + 20\cdot 7}{1\cdot 48 + 20\cdot 1}) = 18.8 \text{ mV}$$

Answer: $\Delta\varphi_{AP} = 18.8$ mV.

Problem 7.5b. The concentration of potassium ions inside the membrane is $[K^+]_I = 350$ mmol·L^{-1}, outside one is in 50 times less, the concentration of sodium ions inside the membrane is $[Na^+]_I = 50$ mmol·L^{-1}, outside one is in 10 times more. Determine the value of resting potential $\Delta\varphi_{rp}$ and action potential $\Delta\varphi_{ap}$, if the cell temperature is $t = 27$ °C.

Data:

Solution:

Answer:____

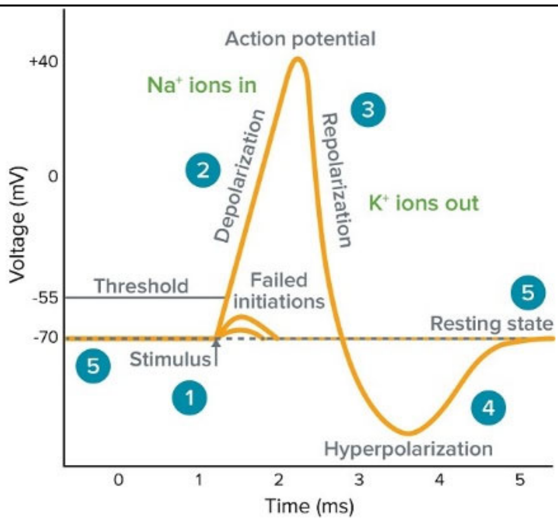


Fig. 7.2b. The action potential formation

Exercise 7.6b. Define the following terms:

Membrane depolarization is ____

____

Reverse of membrane potential is ____

____

Membrane repolarization is ____

____

The current of ions of a certain type can be found from formula:

$$I_i = g_i\left(\varphi_м - \varphi_i^e\right), \qquad (7.5b)$$

where g_i – conductance (the reciprocal of resistance) for defined type of ions (Sm); φ_m – transmembrane potential (V); φ_i^e – Nernst equilibrium potential (V).

Problem 7.7b. With a fixed transmembrane potential φ_m = –40 mV a transmembrane current was detected through a single sodium channel as I_{Na} = 1.6 pA. Calculate the conductance of the channel g_{Na}, if the concentration of sodium ions inside the cell is c_i = 70 mmol·L^{-1}, outside one is c_o = 425 mmol·L^{-1}, the cell temperature is t = 27 °C.

<u>Data</u>:	***Solution:***

Answer:____

Exercise 7.8b. Fill in the table with the main properties of resting and action potentials:

Table 7.1b

The main properties of resting and action potentials

Resting potential	**Action potential**

The Nerve Impulse Propagation Rate

The telegraph equation describes the propagation of the action potential φ as a function of the distance *x* and time *t* over the nerve fiber:

$$\frac{\partial^2 \varphi}{\partial x^2} = \frac{4\rho_a}{D}\left(C_m \frac{\partial \varphi}{\partial t} + \frac{\varphi}{\rho_m l} \right) \tag{7.6b}$$

Exercise 7.9b. Write down the parameters definitions and their units of measurements in the telegraph equation:

D is ____

l is ____

C_m is ____

ρ_a is ____

ρ_m is ____

The solution of this equation has the following form:

$$\varphi = \varphi_0 \exp(-x/\lambda), \tag{7.7b}$$

where φ_0 is the potential in the point $x = 0$ (V); λ is constant fiber length (m).

The constant fiber length is:

$$\lambda = \sqrt{\frac{Dl\rho_m}{4\rho_a}}. \tag{7.8b}$$

Problem 7.10b. At the excitation site of the nerve fiber the transmembrane potential difference is $\Delta\varphi_0$ = 40 mV. Find the difference in potentials $\Delta\varphi$ on the distance x = 50 μm, if the fiber length constant is λ = 70 μm.

Data:	***Solution:***

Answer: ____

Problem 7.11b. Transmembrane potential difference of nerve fibers is reduced in half at the distance x = 30 μm. Calculate the fiber length constant λ.

Data:	***Solution:***

Answer: ____

Problem 7.12b. Specific resistance of membrane thickness unit is ρ_m = 80 mΩ m, resistivity axoplasmic resistance is ρ_a = 0.4 Ω·m, constant fiber length is λ = 4.5 mm. Calculate the axon diameter *D*.

Data:	***Solution:***

Answer: ______________________________

Control questions

1. Ionic mechanism of appearance of rest potential and action potential.
2. Why the nerve impulse can spread along the nerve fiber only in one direction?
3. What is the predominant reason of the membrane resting potential appearance?
4. What determines the spread of excitation along the nerve fiber?
5. How does the permeability for ions relate to the rest potential?
6. The rate of the nerve impulse in the non-myelinated and myelinated fiber is proportional, respectively.......

Individual assignments

1. The concentration of sodium ions in the axoplasm of cuttlefish is $49\ \text{mmol}\cdot\text{L}^{-1}$. What is the concentration of sodium ions in the extracellular medium, if the value of the axon resting potential is 57 mV? Body temperature of cuttlefish is 15 °C.
2. The concentration of potassium ions in the blood of the squid is 16 mmol·L^{-1}. What is the concentration of these ions in the axoplasm of the giant squid axon, if the sea water temperature is 80 °C and the magnitude of the axon rest potential is 79 mV?
3. The concentration of chloride ions inside the cat motor neuron is 9 mmol·L^{-1}, and the concentration of these ions in the extracellular medium is 125 mmol·L^{-1}. Determine the magnitude of the neuron membrane potential if the temperature of the cat's body is 38 °C.
4. Calculate the values of resting potentials of the giant squid axon cells in the upper layers of the ocean, where the temperature is 25 °C, and in the depth ones where the temperature is 6 °C. Concentration of potassium ions in the axon is 410 $\text{mg}\cdot\text{ion}\cdot\text{kg}^{-1}$, and outside the axon is $28\ \text{mg}\cdot\text{ion}\cdot\text{kg}^{-1}$.
5. Transmembrane potential difference at the excitation site is φ_0 = *4 mV,* constant fiber length is λ = 70 μm. At what distance is the potential difference will be *φ = 20 mV?*
6. The constant length of the unmyelinated nerve fiber is 55 μm. The potential at some point of this fiber is equal to φ_0. At what distance from this point the potential will decrease threefold?
7. Calculate the length constant of the unclarified axon with a diameter of 20 μm, if the 1 cm^2 membrane resistivity is 5 kΩ·cm^2, and the 1 cm^3 axoplasm resistivity is – 40 Ω·cm^2.